DOCUMENT RESUME

ED 148 573 SE 023 286

AUTHOR Joyal, Lloyd H.; And Others
TITLE What's Your MQ? Encouraging Creativity in Elementary Mathematics. Revised Edition.
INSTITUTION Wisconsin Univ., Eau Claire.
PUB DATE Sep 77
NOTE 108p.; For related documents, see SE 023 287-291 and ED 129 575; Not available in hard copy due to copyright restrictions; Contains colored pages throughout entire document
AVAILABLE FROM Dr. Juanita Sorenson, University of Wisconsin-Eau Claire, Library 1109, Eau Claire, Wisconsin 54701 ($3.50)

EDRS PRICE MF-$0.83 Plus Postage. HC Not Available from EDRS.
DESCRIPTORS Attitudes; *Creativity; Educational Games; Elementary Education; *Elementary School Mathematics; Instruction; *Instructional Materials; *Mathematical Enrichment; Number Concepts; *Problem Solving; Puzzles
IDENTIFIERS *Wisconsin

ABSTRACT

The need to promote mathematical creativity and suggestions for developing it are discussed in this document. The largest portion of this document consists of a collection of problem sheets, arranged according to 26 content areas. Puzzles, activities, and a variety of problems are included, with an answer key. An annotated bibliography of selected books and materials is also appended. (MS)

**********
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
**********

ED148573

PERMISSION TO REPRODUCE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY
Juanita Sorenson
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) AND USERS OF THE ERIC SYSTEM.

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

# WHAT YOU MUST KNOW

## Entrepreneurship Creativity Business Management

## Acknowledgments

Special thanks should be given to Keith Rogers and Dave Schmidt. These students were enrolled in the Elementary Education Department at the University of Wisconsin-Eau Claire and worked with me during one semester to update and expand this collection of classroom ideas. They are now elementary school teachers in Edgerton and Appleton, Wisconsin, respectively. I especially enjoyed the interaction with these teachers that a curriculum development project of this sort involves.

I am also grateful for the assistance provided by Mrs. Virginia Reece and Ms. Barbara Zadrazil whose expertise in typing, formatting and editing makes this paper useful to classroom teachers.

University of Wisconsin-Eau Claire
Revised September, 1977

LHJ

Lloyd H. Joyal

This product was developed as a part of the ongoing preservice and in-service teacher education program in the Department of Elementary Education at the University of Wisconsin-Eau Claire.

© Competency Based Learning for Individualized Programs (CLIP) 1975

# Contents


| | Page |
|---|---|
| WHAT'S YOUR M.Q.? | 1 |
| APPENDIX A--EXAMPLES OF WEEKLY PROBLEM SHEETS | 9 |
| APPENDIX B--ANSWER KEY FOR WEEKLY PROBLEM SHEETS | 73 |
| APPENDIX C--ANNOTATED BIBLIOGRAPHY | 103 |

# WHAT'S YOUR M.Q.?

Some years ago I became concerned with what I termed "the sterile textbook approach" to the teaching of elementary mathematics. The page-by-page, problem-by-problem style was not, in my opinion, encouraging a favorable view of mathematics by any elementary students.

What could be done about this? How could I change the attitudes of the students toward the study and exploration of mathematics? How could I show them that mathematics could be a lively, challenging, exciting kind of experience - not sterile and uncreative? What techniques would encourage a creative, questioning, explorative style of viewing mathematics?

As I thought about this problem, I felt that, basically, it was a question of supplementing and extending the basic information as presented in the textbook and making some differentiation of assignments to take into account the varying abilities of the students. I wanted to do this in a way that would encourage both the slow and the talented mathematics student. Diversity of thinking and methods of solving problems should be encouraged. The stigma of failure should not be present in the form of grades.

The idea that grew out of this concern was a "mathematics corner" termed "What's Your M.Q.?" This was a weekly series of problem sheets and devices designed to promote the student's diverse mathematical thinking, help him extend textbook learning, and demonstrate that mathematics could be fun. No formal grades were to be given, diverse ways of solving the problems were encouraged, and student participation through individual and small group settings was solicited.

The results were examined in an open class discussion held weekly. Various ways of solving the problem were presented by the students before discussing their particular merits. The brainstorming rule of "no criticism" was in effect.

Initial reaction was slow. Not many students wanted to do "more arithmetic." As they came to see that the weekly sessions were fun and that the views of each pupil were important, the responses grew. Soon pupils were bringing in problems of their own for their classmates to solve and were having fun discussing the responses with them.

Students used these sessions to examine the various solutions proposed and to determine which answers were correct as well as which solutions were the simplest to follow or were solved with the fewest mathematical computations. In other words, they learned from each other. My role as teacher was as a facilitator and guider of learning. I would ask leading questions, give hints, encourage responses, keep a positive attitude toward any suggested solutions.

It was certainly evident to me, as the year progressed, that numbers of my students were enjoying math more and had developed a more positive outlook regarding their daily work. Feedback from many of these students, as they moved into the Junior High program, indicated that the attitudes generated by this approach continued with the student.

At present, I am using this same technique and materials with pre-service elementary teachers. The results are the same. Many of these college juniors indicate that they were turned off to math in the elementary school and hope to use a more positive outlook in their teaching. Comments from these students indicate that a technique such as illustrated here helps them to form a more positive outlook regarding math.

Appendix A will give you several samples of what these weekly problem sheets might look like. In addition to these sheets, crossnumber and magic square puzzles were used, often with pupils contributing their own home-made puzzles. Appendix B is a selected, annotated bibliography of materials to use in helping to make elementary mathematics a creative experience.

What is creativity? You can find almost as many definitions as you can books on the subject. Torrance defines creativity as "the process of sensing problems or gaps in information, forming ideas or hypotheses, testing and modifying these hypotheses, and communicating the results." (13, p. 4) Smith puts it this way: "Creativity is sinking down taps into our past experiences and putting these selected experiences together into new patterns, new ideas or new products." (11, p. 4) Alice Miel believes that the quality of creativity has been shown if an individual makes something new to himself that is satisfying and in that sense useful to him, if he has related things that were previously unrelated in his experience, and if the product is "surprising" to him. She believes that creativity is a quality which each human being is capable of exhibiting. (8, pp. 6-7) All of these opinions, it seems to me, relate directly to the elementary student and his experiences with mathematics.

What school conditions encourage creativity? Prudence Bostwick suggests that inventiveness with time, space, and materials can do much to develop a stimulating environment for learning. The elementary school can manipulate time through the use of such techniques as the unit method of teaching, providing time for browsing (you can't rush a developing idea), and by recognizing the varying rates of learning in each classroom.

Space can be manipulated in the elementary school to provide the best environment for creative learning, including abundant resources for thinking, feeling and acting. The arrangements of rooms and furnishing can vary. Space can be provided for individual projects or interests and wall space can be utilized for displays. Elbow room for experimentation and freedom for thinking should be available for each pupil.

Learning materials may be seen, heard, examined, rejected, accepted, used, discarded, recovered and re-examined for new uses. Materials can be symbols or living things. One should never be surprised at the form that materials take. Most often the problem with materials is that teachers

are willing to accept the obvious, the dull, the usual. We should make an attempt to distinguish between material that is useful for drill or looking up information and material that introduces new values and ideals, stimulates new feelings, and provides needed information at the "impact stage" of creative thinking and doing. (8, pp. 140-163)

Smith places the conditions which tend to develop creativity under five categories: (1) intellectual conditions; (2) physical conditions; (3) social emotional conditions; (4) psychological conditions; and (5) educational conditions. (11, pp. 120-144) He further suggests that these experiences can be natural or contrived. Without going into extreme detail, it might be well to briefly explore each of the five categories suggested, for I feel that they are all important in creating a school setting that will encourage creative expression by the student.

To set intellectual conditions for the emergence of creative thinking and creative development, the teacher should provide opportunities and resources for a great deal of knowledge and a great many skills to be learned through convergent thinking processes. Opportunity should also be provided for pupils to apply divergent thinking processes. Both processes are necessary for the pupil to solve problems adequately. Many open-ended learning situations should be developed utilizing <u>all</u> areas of the curriculum.

Creative development may be greatly enhanced by proper attention to certain physical aspects of the child's environment. These would include room arrangement, availability of materials and the organization of the class in a permissive, creative and democratic atmosphere.

Social-emotional conditions which develop creativity are many:

> (1) rewarding varied kinds of talents and creative achievements; (2) helping children recognize the value of their creative talents; (3) developing creative acceptance of realistic limitations in a problem situation; (4) being sensitive to the needs of children, stressing and praising differences, uniqueness and originality rather than likeness and commonness; (5) accepting "silly" ideas as a sign of creative thinking; (6) helping all children to accept the creative child; (7) developing an atmosphere that is permissive to the extent that children are free to experiment, explore and make mistakes; (8) developing an appreciation for creativity in the classroom; (9) avoiding the equation of difference with mental illness and delinquency; (10) modifying the misplaced emphasis on sex roles; (11) helping highly creative children become less objectionable; (12) helping to reduce the isolation of highly creative children; and (13) helping highly creative children cope with anxieties and fears. (11, p. 130)

Smith suggests that the psychological conditions which appear essential for the development of creativity include:

> (1) a physically and mentally hygienic classroom as described above; (2) a permissive atmosphere based on certain underlying securities; (3) the proper motivation and tensions to agitate creative thinking and creative production; and (4) the proper attitude in the teachers and the school administrator towards creativity. (11, p. 131)

He defines educational conditions as those which develop the reality that creative teaching is a method of teaching different from ordinary teaching. This method is governed by the basic principles of creative teaching culled from research which Smith sets forth as follows:

1. In creative teaching, something new, different or unique results.
2. . . . divergent thinking processes are stressed.
3. . . . motivational tensions are a prerequisite to the creative process. The process serves as a tension-relieving agent.
4. . . . open-ended situations are utilized.
5. . . . there comes a time when the teacher withdraws and children face the unknown themselves.
6. . . . the outcomes are unpredictable.
7. . . . conditions are set which make possible preconscious thinking.
8. . . . students are encouraged to generate and develop their own ideas.
9. . . . differences, uniqueness, individuality, originality are stressed and rewarded.
10. . . . the process is as important as the product.
11. . . . certain conditions must be set to permit creativity to appear.
12. . . . teaching is "success" rather than "failure" oriented.
13. . . . provision is made to learn knowledge and skills but provision is also made to apply these in new problem-solving situations.
14. . . . self-initiated learning is encouraged.
15. . . . skills of constructive criticism and evaluation skills are developed.
16. . . . ideas and objects are manipulated and explored.
17. creative teaching employs democratic processes.
18. . . . methods are used which are unique to the development of creativity. (11, pp. 157-162)

Smith feels that creativity, as such, cannot be taught. However, he feels we can set conditions in the elementary school for it to happen. In creative teaching, then, it is the responsibility of the teacher to

encourage discovery, to develop the capacity of the child to wonder and be puzzled, and to stimulate sensitivity. (11, p. 144)

What are the characteristics of a creative teacher? Shumsky suggests that perhaps the most important single characteristic of the creative teacher is the fact that he or she is a creative person. The creative teacher approaches knowledge with the quesion "What does the subject matter mean to me as a person?" The emphasis is on the teacher experiencing his or her own interpretation of the subject matter and communicating this experience to children. (10, pp. 47-48)

Torrance feels that teachers should value and reward creative achievements in the classroom. He suggests that we need to be respectful of the unusual questions children ask as well as the unusual ideas and solutions they derive. We need to show children that their ideas have value - most of all by just listening to them. We need to provide opportunities and give credit for self-initiated learning. Finally, we need to provide chances for children to think, learn, and discover without the threats of immediate evaluation. (13, p. 16)

Alice Miel suggests that the creative teacher can be judged by the quality of opportunities actually provided by the teacher for his or her pupils. (8, p. 9)

There is no question in my mind, or in those of authorities, that the school setting and the outlook of the teacher play important roles in encouraging creative expression in elementary students. But what about mathematics in particular? Shumsky cites two major forces as being conducive to the creative orientation of elementary mathematics, namely: (1) emphasis on the inductive process of discovery through manipulation; and (2) emphasis on intellectual playfulness which promotes insight into the rationale of the structure of arithmetic. (10, p. 157)

Robert B. Davis, one of the originators of the Madison Project in elementary mathematics, would agree that the discovery approach is important in encouraging interest and creativity in mathematics. He would want the student to have experience in recognizing and dealing with open-ended situations and in extending open-ended situations by original creative work. (9, pp. 126-127)

Burton, Kimball and Wing strongly suggest that mathematics teaching should provide the student with the opportunity to discover the facts and relationships of mathematics. They feel that educational research has pointed this out very clearly and that "this is no longer a matter of opinion." In accepting this point of view, they suggest several principles to follow in encouraging divergent thinking and discovery:

1. Mathematical relationships take on more meaning when they are discovered by the student.

3. Students will better understand the nature of proof if this concept is developed slowly, building on that which is already familiar to the pupil.

6. Mathematics should be taught so as to emphasize original and independent thinking. Particular care must be taken to avoid providing the sort of cues and organizing the original exercises in such a way that the subject is reduced to a "bag of tricks." Specific techniques of analysis should be identified, developed, used, and generalized.

7. The materials and methods of mathematics must stimulate the curiosity and interest of each student so that he will carry his study of the subject to a point compatible with his ability.

(2, pp. 393-394)

In reporting on a creative mathematics project, Mary Laycock says creative mathematics is "the ability to analyze a given problem in many ways, observe patterns, see likenesses and differences, and on the basis of what has worked in similar situations decide upon a method of attack in an unfamiliar situation." (6, p. 325) She goes on to describe the work in the classroom using Cuisenaire rods and other manipulative material, much of it "created" by the children. She describes eight year olds making icosahedrons and dodecahedrons and enjoying it. She concludes by saying:

> "The teacher is the key. He must not be afraid of this kind of mathematics. If he loves mathematics and gets excited about it, the sky is the limit." (6, p. 328)

Both Golden (4, p. 113) and Cochran (3, p. 215) stress the importance of the attitude of the teacher in fostering creativity in elementary school mathematics. These reports on action research in the classroom conclude that we must listen to the ideas of children, let them try out their ideas and pursue them to a reasonable outcome.

These articles described original, exciting work done by children in the first and second grades. I would very much like to observe these students at a later age if this sort of enthusiasm could be maintained. This, I think, is the real key to the creative adolescent and adult - early encouragement and receptivity to original ideas.

Bachtold and Werner discussed an eight month training program in creative thinking for 17 fifth and sixth graders. Assignments and test questions were designed to elicit divergent and evaluative thinking. Pre and post tests were administered using the Minnesota Tests of Creativity, Verbal Form A.

Results suggest further research, especially since the validity and

stability of most "creative tests" have not yet been satisfactorily established. The authors also suggested that the effectiveness of creativity teaching be further tested. (1)

By far the most exhaustive reference available is Wescott and Smith's Creative Teaching of Mathematics in the Elementary School. This two-hundred-plus page book describes in detail the authors' point of view in regard to the creative teaching of elementary mathematics and also suggests numerous ways to creatively stimulate a mathematics program - from addition to modular arithmetic. They feel that creativity in mathematics is the birth or discovery of new number patterns; the development of new relationships between previously known mathematical ideas; the modification and/or variance of existing patterns, symbols and relationships; the discovery of new applications of mathematical ideas. (15, p. 210)

A quote from William B. Michael's Teaching for Creative Endeavor seems an appropriate conclusion.

> Creativity comes in some odd shapes and parents have to be willing to recognize this. Civilization in its desire for conformity tends to put roadblocks in the way of the creative individual, and if we are not careful we tend to class the creative person with the disturber and the delinquent. We need in our society, in order for it to live, a public which can appreciate creativity and will reward it rather than vote it out of existence.
>
> Thus, it would appear that creative children are more like adults than other children; they tend to want more autonomy and more independence. All able people are pretty reasonable about limits and restraints; what no one wants, including creative children, are arbitrary limits. E. Paul Torrance, one of the great writers in the field of creativity in the country today, once answered the question, "What makes a child more creative?" by "Anything which makes him more free and alive." Without freedom no one can be creative. And creativity is nurtured by warmth and affection. All of us, and especially children, tend to create for those we love. (7, p. 341)

BIBLIOGRAPHY


1. Bachtold, Louise M. and Emmy E. Werner. "An Evaluation of Teaching Creative Skills to Gifted Students in Grades 5 and 6," The Journal of Educational Research, 63: 253-256, February, 1970.

2. Burton, William H., Roland B. Kimball, and Richard L. Wing. Education for Effective Thinking. New York: Appleton-Century-Crofts, Inc., 1960.

3. Cochran, Beryl S., Alan Barson, and Robert B. Davis. "Child-Created Mathematics," The Arithmetic Teacher, 17: 211-215, March, 1970.

4. Golden, Sarah R. "Fostering Enthusiasm Through Child-Created Games," The Arithmetic Teacher, 17: 111-115, February, 1970.

5. Gowan, John Curtis, George D. Demos, and E. Paul Torrance. Creativity: Its Educational Implications. New York: John Wiley & Sons, Inc., 1967.

6. Laycock, Mary. "Creative Mathematics at Nueva," The Arithmetic Teacher, 17: 325-328, April, 1970.

7. Michael, William B. (ed). Teaching for Creative Endeavor. Bloomington: Indiana University Press, 1968.

8. Miel, Alice (ed.). Creativity in Teaching. Belmont, California: Wadsworth Publishing Co., Inc., 1961.

9. Shulman, Lee S. and Evan R. Keisler (eds.). Learning By Discovery. Chicago: Rand McNally & Co., 1966.

10. Shumsky, Abraham. Creative Teaching in the Elementary School. New York: Appleton-Century-Crofts, 1965.

11. Smith, James A. Setting Conditions for Creative Teaching in the Elementary School. Boston: Allyn and Bacon, 1966.

12. Sullivan, John J. and Calvin W. Taylor. Learning and Creativity. Washington, D.C.: National Science Teachers Association, 1967.

13. Torrance, E. Paul. Creativity. Washington, D.C.: National Education Association, 1963.

14. Weils, Peter. "Creating Mathematics with a Geoboard," The Arithmetic Teacher, 17: 347-349, April, 1970.

15. Westcott, Alvin M. and James A. Smith. Creative Teaching of Mathematics in the Elementary School. Boston: Allyn and Bacon, Inc., 1967.

# APPENDIX A

## EXAMPLES OF WEEKLY PROBLEM SHEETS

## EXPLANATION OF PROBLEM CODE

The following problems are arranged according to their content or the content that they seemed best adapted to. The code that precedes each problem refers to the content of the problem and the page number of the source. There is an abbreviated bibliography at the bottom of each page. In addition, there is an annotated bibliography at the end of the book.

The problems contained in this book have not been leveled. It is the responsibility of the teacher to determine appropriate problems and use them effectively. The content levels have been adapted from the "Code for Hierarchy of Mathematics Content Objectives" as it appears in the paper entitled, Individualizing Mathematical Learning in the Elementary School.

A sample problem will look like this:

D-166 Can you make 1000, using only eight 8's?

Content (Addition)

Page Number of Source

The content areas are listed below:

| | | |
|---|---|---|
| A sets | K integers | V measurement |
| B counting | L writing sentences | W number line |
| C numeration | M solving sentences | X constructions |
| D addition | N ratios | Y ordered pairs |
| E subtraction | P 3-dimensional figures | Z probability and statistics |
| F multiplication | Q 2-dimensional figures | AA decimals |
| G division | R similarity and congruence | AB percents |
| H fractions | S symmetry | AC logic |
| J number theory and order | T "money" and "time" | |

A-56 TOM'S CLASS

Tom's class had been questioned on which subjects they liked and disliked, and the results when tabulated were as follows:

18 liked Mathematics
32 liked English
25 liked Languages

Sixteen said they liked both English and Languages but only 7 said they liked both Mathematics and Languages, and only 8 said they like both Mathematics and English. Only 3 said they liked all three subjects and none admitted that he did not like any subject.

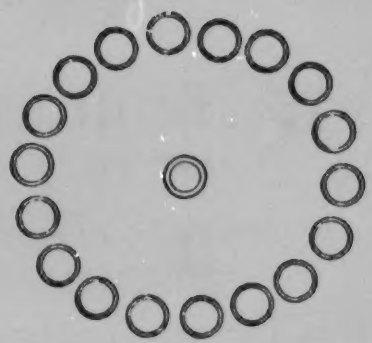
How many children are there in Tom's class?

B-56 What letter follows OTTFFSSE__?

Longley-Cook, L.H. Fun with Brain Puzzlers, p. 56.

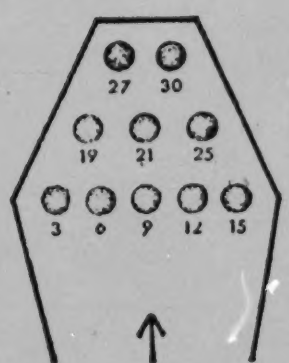
Litton Industries. The Best of Problematical Recreations, Volume 1, p. 56

D-111 NUMBER PLATE



Place the numbers 1 through 19 in the appropriate circles so that three numbers in a straight line will total 30.

D-127 WINNING WAYS



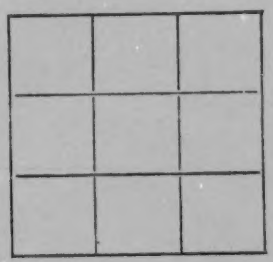

It looks so simple...but is it?

This drawing shows the layout of a fairground game, and the idea is to roll three balls down the runway into the numbered holes.

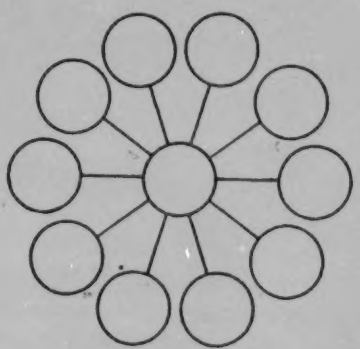
To get a prize, you have to score exactly 50. What is the winning combination?

Doubleday, Eric. Test Your Wits, Vol. III, pp. 111 & 127

D-10 In this magic square you must use the digits 1 through 9, each one once only. Make a sum of 15 in all directions--horizontally, vertically, and diagonally. (There are many solutions.)

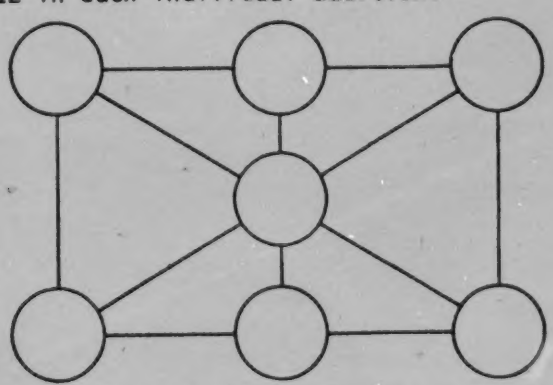


D-10 Take the digits 1 through 11 and arrange them in the circles shown, so that they add to the same result in all directions. (Add the 3 circles together on the same straight line. The sum should equal all the other straight line totals.)



D-11 If a farmer had 39 haystacks in one corner of a field, 15 haystacks in another corner, 10 in another, and 5 in the last, how many haystacks would there be altogether?

D-11 Using the numbers 1 through 7, place a different number in each circle in such a way that all connecting lines containing three of the numbers add up to 12. The sum of all straight lines must be 12 in each individual addition.



Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp.10-11

D-12 A MAGIC SQUARE

"Magic squares" have intrigued people for thousands of years.

The simplest "magic square," which was known to the ancient Chinese at least three thousand years ago, used the nine numerals as shown in this diagram:

| | | |
|---|---|---|
| 8 | 3 | 4 |
| 1 | 5 | 9 |
| 6 | 7 | 2 |

This is called a "magic square" because all rows and columns and the two main diagonals as well, add up to the same total: in this case 15, which is the "magic constant" of this example.

It is not an essential condition, however, for the numbers in a "magic square" to be consecutive.

We now show the outline of a different nine-number "magic square," with only two of the individual numbers filled in: 7 and 13. For this "magic square" the "magic constant" is 111, and you have to fill in the other seven numbers.

| | | |
|---|---|---|
| | | 7 |
| 13 | | |
| | | |

D-41 THERE'S NO LOGIC IN THIS

Write down the nine digits, 1 through 9, in their regular order, preferably with spaces between them. Then write in two plus signs and two minus signs, so that the resulting expression equals 100.

For example, if you had to make up 24 using only the first three digits and one plus sign, you could have 1 + 23 = 24.

D-166* Can you make 1000, using only eight 8's?

Hunter, J. A. H. Hunter's Math Brain Teasers, p. 12 & 41

Meyer, Jerome S. Fun With Mathematics, p. 166

D-127 FINDING A PERSON'S AGE

If the person is <u>older</u> than you:

1) Write down a number containing as many nines as there are digits in your age.

2) Subtract your age.

3) Ask the other person to add the result obtained in 2) to his age without letting you see it.

4) Tell the other person to cancel the left-hand digit and add it to the remaining digits.

5) Ask the other person to tell you the result in 4), and you can immediately reveal his age.

Assume that you are 19; then write down 99 and subtract 19, which leaves 80. Suppose the other person is 21, then he finds the sum 80 + 21 = 101. Cancel the left-hand digit 1 and add it to 01; thus, 01 + 1 = 2. Then let him tell you that his result is 2 and you will immediately reveal that his age is 21.

If the other person is <u>younger</u> than you, add your age (in step 2) and ask him to subtract his age in step 3). Thus, if you are 23, write 99 + 23 = 112. Suppose he is 19, then 112 - 19 = 103. Cancel the left-hand digit 1 and add it to 03; thus, 03 + 1 = 4. Let him tell you that his result is 4 and you will immediately reveal his age is 19.

D-133 GUESSING THE THROW WITH TWO DICE #1

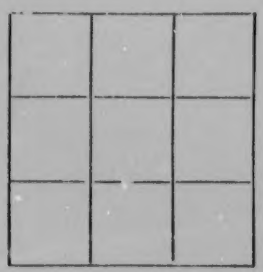
While my back is turned throw two dice on a table and keep in mind the two numbers that turn up. Select any one of these numbers, multiply it by 5, add 7 to the product, multiply this sum by 2, and, finally, add to this result the other number that turned up. Tell me this final sum and I shall tell you the two numbers originally thrown.

Suppose you throw

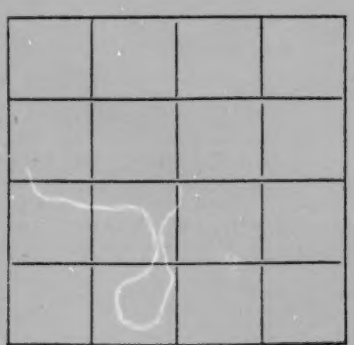
Now suppose you select 3, multiply 3 x 5 = 15, add 7 to the product 15 + 7 = 22, and multiply this sum by 2; thus, 22 x 2 = 44. Finally, add to this product the other number 5 that turned up to get 44 + 5 = 49. Tell me this final sum 49 and I shall reveal that you originally threw a 3 and a 5. If you can figure out the solution to this problem, you will then be able to try your new found magic on your friends.

Mira, Julio, A. Mathematical Teasers, pp. 127 & 133.

D-12 Using the numbers 1, 2, 3, 10, 11, 12, 19, 20, and 21 only once, place them in the squares in such a way that the sum of all horizontal, vertical, and diagonal lines is 33.

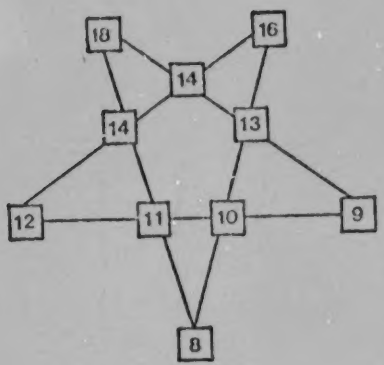


D--15 How may the first 16 digits be arranged so that the sums of the vertical, the horizontal, and the two diagonal rows equal 34? (There are many different solutions.)



Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 12 & 15

D-101 TREASURE TROVE




Strange though it might seem, this is a map of the fortifications guarding a sultan's treasure--five forts around the perimeter, and five more inside.

The figures inside each represent the number of men in the various forts.

For some reason, the sultan wanted to redeploy his men so that in each of the five lines of four forts there were fifty men.

However, none of the garrisons could be split up. Each group of men, in other words, had to be moved in one body.

Can you show how the sultan redeployed his troops?

Doubleday, Eric. Test Your Wits, Vol. III, p. 101

E-27 (a). Two men in the Army had a potato-peeling contest to see who could peel the most potatoes in a given time. One peeled two hundred thirty-six potatoes, and the other peeled three hundred and won. How many potatoes did both peel altogether? (This problem is more effective if you state it orally to your friends.)

(b). If you have twenty (six or sick) sheep, and one dies, how many are left? (Must be posed orally.)

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, p. 27

E-121 If you can solve the reason for the next three problems then you
E-121 can be the magician and try them on your friends.
E-123

E-121 REVEALING THE DIFFERENCE BETWEEN DIGITS

Write a number containing two digits without letting me see it. Form a new number by reversing the order of the digits and find the difference between the two numbers. Tell me the difference between the two numbers and I shall tell you the difference between the digits of the number you wrote.

Suppose you write 91, obtain 19 by reversing the order of the digits, and tell me the difference 91 - 19= 72. I shall immediately reveal that the difference between the two digits of the number that you wrote is 8.

E-121 TELEPATHY # 1

Without letting me see it, write a number of four digits such that the digits decrease by one from left to right. Reverse the order of the digits, obtaining a new number and subtract this number from the original one. Then I, the wizard, can read your mind and, without seeing your work, I can tell you what the difference is.

Suppose you write 7654, reverse the digits to form a new number 4567, and find the difference 7654 - 4567 = 3087. Without seeing any of your work, I can read your mind and reveal that the difference is 3087.

E-123 TELEPATHY # 2

Without letting anyone see it, write any number of three digits such that the digits decrease by one from left to right. Reverse the order of the digits and obtain a new number, then subtract this number from the original one. Now I, the wizard, can read your mind and, without seeing your work, I can tell you what the difference is.

Suppose you write 765, reverse the digits to form a new number 567, and find the difference 765 - 567 = 198. Without seeing any of your work I can read your mind and disclose that the difference is 198.

Mira, Julio A. Mathematical Teasers, pp. 121 & 123

E-133 COUNTER CHARM

The jeweller laid out 33 diamonds in a semi-circle on the counter.

"What a beautiful necklace they'd make," he told his customer. "And it's worth exactly $65,000."

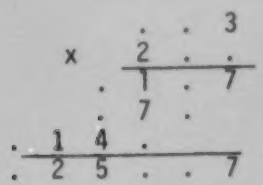
The center diamond, of course, was the biggest and the most valuable. The one immediately to its left was worth $100 less - and each successive stone diminished in value by $100.

On the right of the larger stone, however, each diamond's value diminished by $150.

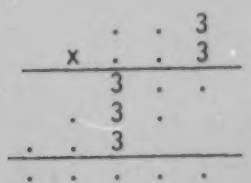
What was the value of the large diamond?

Doubleday, Eric. Test Your Wits, Vol. III, p. 133

F-35 This is a multiplication problem in which some numbers are omitted. Can you write in the missing numbers?



F-36 Figure out the missing numbers:



F-16 SCHOOL NOTES

If 6 boys fill 6 notebooks in 6 weeks and 4 girls fill 4 notebooks in 4 weeks, how many notebooks will a class of 12 boys and 12 girls fill in 12 weeks?

F-7 FENCE ME IN

I plan to fence my frontage on to the main road, which is 100 feet long. Fencing costs $5.00 per 10 feet and posts, which are to be 10 feet apart, cost $5.00 each. What will be the total cost of the fence?

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 35-36

Longley-Cook, J.H. Fun with Brain Puzzlers, p. 16

Longley-Cook, L. H. Work This One Out, p. 7

F-125
F-124

These two problems are written as if you were telling them to someone else to help you when you want to surprise them with your trick. But before you try it out on a friend you must solve the trick yourself.

F-125 REVEALING A NUMBER #1

Write any integer, or whole number, on a piece of paper without letting me see what you have written. Square this integer; then square the next larger integer. Tell me the difference between these squares and I shall tell you the number that you originally selected.

Suppose you write 11, square 11 to obtain 121, and square 12 to get 144. Tell me the difference 144 - 121 = 23 and I shall immediately tell you that the number you selected is 11.

F-124 TELEPATHY #4

Without letting me see it, write any number of four digits such that the digits decrease from left to right, form a new number by reversing the order of the digits, and subtract it from the original number, finding the difference; then form another new number by reversing the order of the digits in the difference and add this number to the previous difference. Without seeing your work I can read your mind and tell you the result.

Suppose you write 8643, form 3468 by reversing the order of the digits, and find the difference 8643 - 3468 = 5175. Now form another new number 5715 by reversing the order of the digits in the difference and find the sum 5175 + 5715 = 10890. Without seeing any of your work, I can read your mind and tell you the result is 10890.

Mira, Julio A. Mathematical Teasers, pp. 124-125

F-19 TAKE A GUESS

Here's a little problem which has probably caused more scratching of heads than almost any other.

You are not, incidentally, allowed to write down any calculations . ... what's needed is sheer guesswork.

You're asked to imagine that you have a pile of paper, each page one hundredth of an inch thick.

First you put down one sheet, then double what you have--putting down two sheets. Next, you put down four sheets, and afterwards eight. And so on.

Quite simply, if you double the number of sheets laid down thirty times in all, how high, roughly, will your pile of paper be? A thousand feet, a hundred feet, a foot . . . or what?

Doubleday, Eric. Test Your Wits, Vol. III, p. 19

G-115 MAKING A NUMBER DIVISIBLE BY THREE

Write a number of as many digits as you please and I shall name a single digit that may be placed before, after, or anywhere within the number making it exactly divisible by 3. Suppose you write 765932. Then I shall put a 1 before, after, or anywhere within the number so that the new number is exactly divisible by 3. Thus, 1765932, 7651932, 7659321, etc. are all exactly divisible by 3.

G-115 MAKING A NUMBER DIVISIBLE BY NINE

Write a number of as many digits as you please and I shall name a single digit that may be placed before, after, or anywhere within the number making it exactly divible by 9. Suppose you write 765932, then I shall put a 4 before, after, or anywhere within the number so that the new number is exactly divisible by 9. Thus, 4765932, 7645932, 7659324, etc. are all exactly divisible by 9.

G-116 MAKING A NUMBER DIVISIBLE BY ELEVEN

Write a number consisting of any number of digits and I shall name an integer that may be added to (or subtracted from) this number that makes it exactly divisible by 11. Suppose you write 765932, then I shall add 9 to this number; thus, 765932 + 9 = 765941 which is exactly divisible by 11. If you write 639527, then I shall subtract 9 from this number; thus 639527 - 9 = 639518 which is exactly divisible by 11.

G-120 REVEALING THE REMAINDER

Write any number without letting me see it. Divide this number by 9 and tell me the remainder. Multiply the original number by a digit which I name and divide the resulting product by 9. Then I shall tell you the remainder.

Suppose you write 134 and divide by 9 obtaining the remainder 8. Then multiply 134 by the digit 7, which I name, obtaining the product 938 and divide this product by 9. Without seeing your work, I shall reveal that the remainder is 2.

Mira, Julio A. Mathematical Teasers, pp. 86, 115-116 & 120

G-58 If it takes one minute to make each cut, how long will it take to cut a 10-foot pole into 10 equal pieces?

G-60 1 2 3 4 5 6 7 8 = 9. Using all digits from 1 to 8 and all 4 processes--addition, subtraction, multiplication, and division--get a result of 9. The digits 1 to 8 must be used exactly as appear above, and one operational sign must be put between each digit. Example:

1+2x3÷4x5+6-7x8=9

This is wrong, of course, because the answer is not 9. To get the answer, the problem must be worked in the order in which the digits appear rather than by the correct rules of algebra.

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp.58 & 60

H-78 A man passed 1/6 of his life in childhood, 1/12 in youth, and 1/7 in bachelorhood. Five years after he was married, a son was born who died 4 years before his father, at half the age at which his father died. What was the father's age when he died? Rembering how to change denominators to add fractions will help you with this problem.

H-79 If a boy and a half can eat a pie and a half in a day and a half, how long will it take 18 boys to eat 18 pies? (Half a a boy is a small boy.)

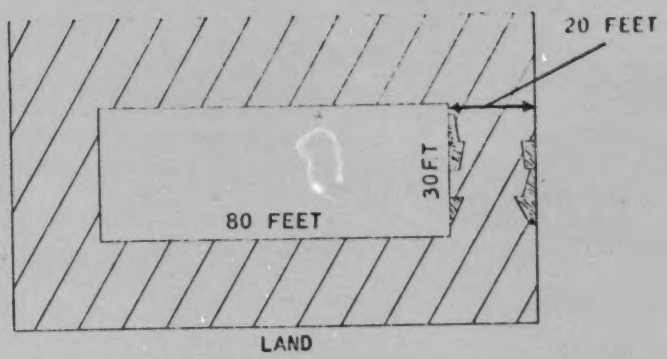
H-79 Two mothers and two daughters went shopping. After totaling all their purchases, they found they had spent $18 altogether. Yet each, spent the same amount and each spent exactly 1/3 of the total. How was this possible? Think of the different combinations of people that could be mothers and daughters when doing this problem.

H-81 A farmer told two boys they could take apples off his tree, provided neither took more than 20 apples. After a minute, one boy asked, "Have you picked your limit yet?" The other replied, "Not yet, but if I had twice as many as I have now, plus half as many as I have now, I would have the limit." How many did he have?

H-76 Two volumes of an encyclopedia are standing side by side, in order, on a bookshelf, Volume I to the left of Volume II. A bookworm eats a hole, starting at page 1 of Volume I, and eats his way in a straight line to the last page of Volume II. If each cover is 1/8 of an inch thick, and each book without the cover is 1 inch thick, how far does the bookworm travel?

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 76, 78, 79 & 81

H-76 There is a castle surrounded by water, as shown below.




During a storm, the bridge from the land to the castle is washed away. The distance across the water from the land to the castle is 20 feet. A man desiring to cross to the castle found two long boards. One was 19½ feet long and the other was 18 feet long. This posed a problem: How was he to get across the 20 feet of water? His 19½-foot board was not long enough; he could not nail the boards together. He finally figured a way to get across the water. How did he do it?

H-77 When Mr. Einstein was asked how many students he had, he replied, "One-half of them study mathematics, one-third of them study geometry, one-seventh of them study chemistry, and there are twenty who do not study at all." How many students did Mr. Einstein have?

H-88 HOMEWORK

A teacher assigned homework to his class and told the students that on each day after the first, they must do twice the number of problems that they had done so far. If at the end of five days, the students had completed one third of the problems, how long will it take them to do all of their problems?

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 76-77

Mira, Julio A. Mathematical Teasers, p. 88

J-14 SOME SERIES

Find the rule for each of these series and write down what the next term will be.

| | | | | | | |
|---|---|---|---|---|---|---|
| (1) | 8 | 11 | 14 | 17 | 20 | 23 |
| (2) | 6 | 9 | 18 | 21 | 42 | 45 |
| (3) | 12 | 9 | 3 | 6 | 3 | 1 |

J-45 Supply the missing number in the following sequence:

10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 24, ____, 100, 121, 10,000.

Longley-Cook, L. H. Fun With Brain Puzzlers, p. 14

Litton Industries, Inc. The Best of Problematical Recreations, Volume I, p. 45

## INTEGERS

K-11 A GROUP OF NUMBERS

Find the largest group of different positive integers less than 100 such that no combination of them added together totals 100.

K-50 AN UNUSUAL SERIES

Can you find the fifth term in the following series?

77, 39, 36, 18, _____

K-12 LETTERS FOR DIGITS

In the following subtraction problem each digit has been replaced by a certain letter. Can you reconstruct the original problem?

```
  A B A
-   C A
-------
  A B
```

K-77 TWO-DIGIT NUMBERS

(a) What number is twice the product of its digits?
(b) What number is three times the sum of its digits?
(c) What number is the square of its units digit?
(d) What number exceeds its reversal by 20%?
(e) What numbers plus their reversals sum to perfect squares?

K-16 ADDING UP TO 100

Take any integer (1 to 9). Write it down four times, and connect up by arithmetical signs so that the expression will equal 100.

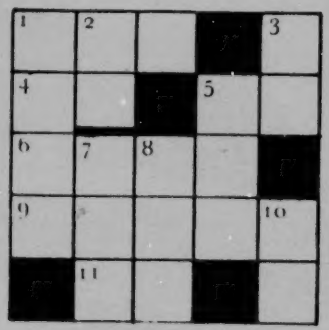

Friedland, Aaron J. Puzzles in Math & Logic, p. 11

Longley-Cook, L. H. Fun With Brain Puzzlers, pp. 12 & 50

Mott-Smith, Geoffrey. Mathematical Puzzles, p. 77

Abraham, R. M. Diversions and Pastimes, p. 16

K-61 STRAIGHT AS A DIE



(There are no 0's)

Across

1. A multiple of 11 across.
4. A prime number.
5. Odd.
6. Digits all different; their sum is 10.
9. Digits all different.
11. A prime number. The sum of the digits is more than 5 and less than 11.

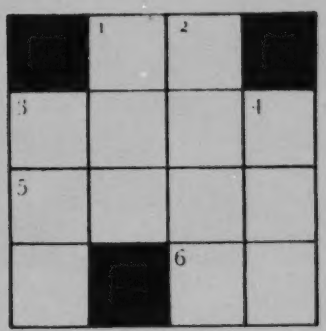
Down

1. Odd. Each digit is greater than the preceding one.
2. Sum of digits is greater than 11.
3. The same two digits as in 4 across.
5. A perfect cube.
7. Even. Sum of digits is 19.
8. A perfect square.
10. A perfect cube.

Emmet, E. R. Puzzles for Pleasure, p. 61

K-62 STRAIGHT AS A CORK-SCREW

(There are no 0's)



Across

1. Sum of digits is 13.
3. Each number is two different than the one in front of it.
5. Digits all even; their sum is 12.
6. A square.

Down

1. The sum of the digits is the same as the sum of the digits of 4 down.
2. The sum of the first two digits is the same as the sum of the last two digits of 1 down.
3. A cube.
4. The cube of an odd number.

Emmet, E. R. Puzzles for Pleasure, p. 62

L-4 THE LUCKY FIND

Jill's father looked at the money on the table. "You've got quite a lot there," he remarked.

"I found two dollars on the sidewalk," the child explained.

"You were lucky." Her father laughed. "Now you've got just five times as much as you'd have had if you'd lost two bucks."

How much did Jill have before her lucky find?

L-7 WHO CHECKS THE CHECKS?

"Separate checks, please," Joe told the waitress. "Sam had as many coffees as the total number of items Ted and I have each consumed."

The girl handed each his own check. "It still comes to one dollar and five cents for each of you," she said. "One check would have done."

"Quite a coincidence then," chuckled Sam. "We all had coffee, pie, and ice cream and nothing else. But different numbers of ice creams, and Ted ate the most."

It was certainly odd the way those checks came out. Coffee was a dime a cup, ice creams were twenty cents each, and pie fifteen cents a piece.

What exactly had each of them had?

L-28 THE CROCODILE

That croc was a freak,
  not at all the right shape,
With a tail thrice as long
          as its head.
Its body was short,
  far too short for the rest;
Half as long as its tail,
          so they said.

The body and tail
  measured thirteen feet six:
An ugly great brute,
          you'll agree.
But I never discovered
  the length of its head.
Just how long do you think
          it would be?

Hunter, J. A. H. Hunter's Math Brain Teasers, pp. 4, 7 & 28

L-13 HOW OLD IS MY DAUGHTER?

My daughter is twice as old as my son and half as old as I am. In twenty-two years my son will be half my age. How old is my daughter?

L-15 AGES

Bill and his brother Jim have combined ages of 30. In fourteen years Jim will be three times the age Bill is now. How old is Jim?

L/M-10 DISTANCE PROBLEM

Percy Poppon had an appointment in London at two o'clock in the afternoon. If he left his home and travelled at 15 miles per hour, he would arrive at one o'clock, and if he travelled at 10 miles per hour he would not arrive until three o'clock. What was the distance from his home to London?

L/M-3 Three men play a game with the understanding that the loser is to double the money of the other two. After three games, each has lost just once; and each has $24. How much did each have to start?

L-28 Three men entered a hotel and asked for a room, but the clerk said there was only one available but that he would put dividers in the room. The room would cost $30. Each man paid $10. After the clerk thought the situation over, he decided that he had overcharged the men and called the bellboy and instructed him, "Take this five dollars back to the three men. Tell them I overcharged them and that they should divide this five dollars among themselves." The bellboy thought that dividing $5 among three men would be fairly difficult and, being dishonest, kept $2 for himself. Then he returned $1 to each man so, actually, the cost to each man was $9. Now, 3 times 9 is 27 and the $2 the bellboy kept makes $29. What happened to the other dollar?

Longley-Cook, L. H. Fun With Brain Puzzlers, pp. 13 & 15.

Abraham, R. M. Diversions and Pastimes, p. 10.

Litton Industries, Inc. The Best of Problematical Recreations, Volume I, p. 3.

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, p. 28.

L-66 A BALLAD OF BUTTON

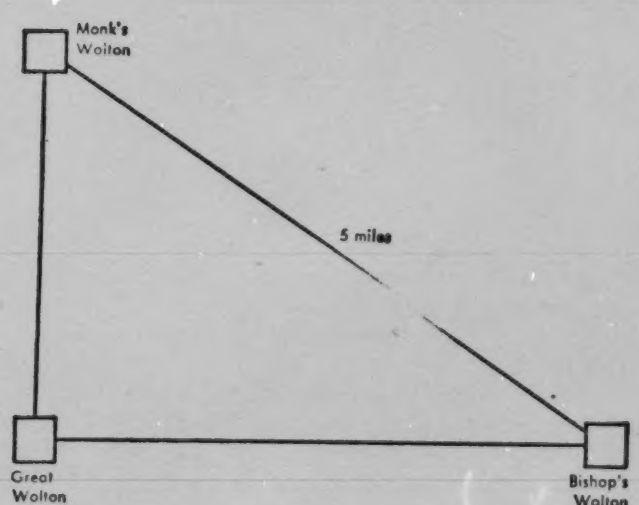
A beery old boozer
called Button
Ate ninety-four cents'
worth of mutton.
Each cutlet cost seven,
Each chop cost eleven.
How much did he eat,
that old glutton?

L-47 THE WOLTONS

"The English countryside, with its little hills and dales and fascinating village names, and the English bicycle, which is such a pleasure to ride, lend themselves to the construction of puzzles," said Old Prime.

"Monk's Wolton is due north and Bishop's Wolton is due east of Great Wolton, and the road to each from Great Wolton is straight.

"Starting from Great Wolton it takes me the same time to bicycle to Monk's Wolton and back as it does to bicycle to Bishop's Wolton and back, but then it is level all the way to Monk's Wolton and a steady uphill climb to Bishop's Wolton. Uphill I bicycle at half my level speed and downhill I bicycle half as fast again as I do on the level. If it is 5 miles as the crow flies from Monk's Wolton to Bishop's Wolton, how far is Monk's Wolton from Great Wolton?"




Hunter, J. A. H. Hunter's Math Brain Teasers, p. 66

Longley-Cook, L. H. Work This One Out, p. 47

L-29 THOSE WERE THE DAYS

Gil Giles is the biggest farmer around here, but he is never tired of telling how he started. "I was a hired hand over with Sam Miller's dad until I married Mrs. Giles," he'll say, chewing the end of a 50-cent cigar, "and then I rented a little place and used my savings to get some stock."

Mostly he pauses at that point to make sure he has your attention. And then he'll go on: "Things were very different in those days. I bought pigs at nine dollars, geese at seventy cents, and chickens at fifty cents: a hundred head altogether and they cost me just a hundred dollars."

Well, he's certainly worked hard to get where he is now. But can you figure out how many geese he bought?

L-17 ON THE CARDS

Just two golf balls are on the edge of the last green - and the fact that they are so close together indicates how tight the match was.

In fact, the two men were level coming to the eighteenth. But Jones took one putt less than Smith.

The digits in Smith's final score were the same as his total for nine holes--but reversed. And that total was 10 shots more than if he had taken three strokes for each hole.

What was Jones's score for the round?

Hunter, J. A. H. Fun With Figures, p. 29

Doubleday, Eric. Test Your Wits, Vol. III, p. 17

M-25 IN THE BAG

Imagine, if you will, a conversation between a horse and a donkey who are struggling along, each with a load of sacks.

"Why are you complaining?" asked the horse. "If you gave me one sack, I'd have twice as many as you, but if I gave you one of my sacks we'd still only have equal loads."

How many sacks did each have?

M-39 THE PASSING YEARS

Jack is exactly two and a half times as old as his sister, Jill.

The curious thing, though, is that last year he was three times as old--and the year before that, four times as old.

Three years ago, he was seven times as old.

Without using pencil or paper, can you work out the ages of the two children?

Doubleday, Eric. Test Your Wits, Vol. III, pp. 25, 39

M-9 THE RAINS CAME

In a certain tropical town ten and a half inches of rain fell in a week and a half. What was the average daily rainfall over the period?

Longley-Cook, L. H. Work This One Out, p. 9

## RATIOS

N-3 HOW HIGH IS A POLE?

How high is a pole that casts a shadow 21 feet long, if a 6-foot man standing beside the pole casts a shadow 4½ feet long?

How deep is a well, if a rope that just reaches from bottom to top can be wrapped exactly 12 times around the cylindrical drum of a windlass, the drum being 7 inches in diameter?

How many sheep jump over a fence in an hour if 10 sheep jump over a fence in 10 minutes?

N-15 MOPPING UP

If 7 maids with 7 mops sweep 7 tons of sand in 7 months how long does it take 10 maids with 10 mops to sweep 10 tons of sand?

N-15 FOUR APPLES APIECE

"You didn't tell me you were going," cried Sally, as her two brothers came in with their apples. "I wanted some too."

Paul put his bag on the table. "Okay, you'll have your share," he told her. "Jim got five and I got seven, so we can have four each."

"And they'll cost you twelve cents in all, the same as the price we paid." added Jim.

How did the boys split up that twelve cents.

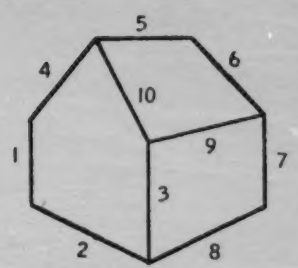
Mott-Smith, Geoffrey. Mathematical Puzzles, p. 3.

Longley-Cook, L. H. Work This One Out, p. 15

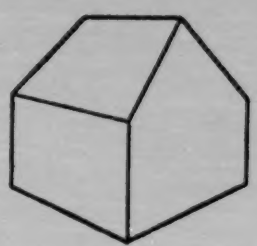
Hunter, J. A. H. Hunter's Math Brain Teasers, p. 15

P-127 Here is a picture of a house which faces to the left. Can you change this house so it faces to the right by moving only two of the lines? Tell what numbered lines you would move, and where you would place them.

House facing left



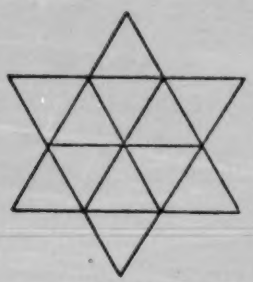

House facing right



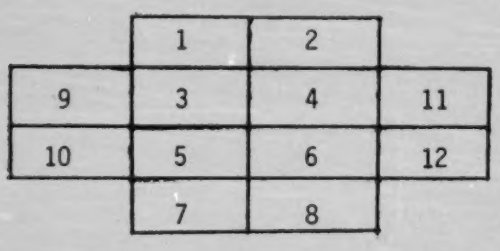
Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, p. 127

## 2-DIMENSIONAL FIGURES

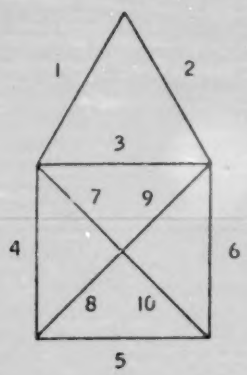
Q-119 How many triangles of any size are in this star?



Q-120 How many rectangles are in this figure? (The numbers represent the 12 small rectangles and these numbers will be used as an aid in identifying the different rectangles.)




Q-125 Retrace this pattern in one continuous line, without lifting your pencil from the paper. (The numbers shown on the lines are merely for the purpose of determining the correctness of the answer, or for following a particular path described by an individual.)




Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 119, 120 & 125

Q-126 Using 8 straight lines, how can you make 4 triangles and 2 squares?

Q-132 Using 9 straight lines of exactly equal length, can you form 5 equilateral triangles?

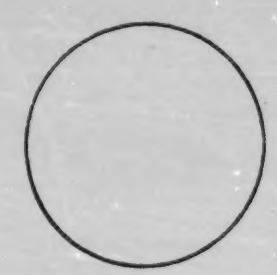
Q-4 HEXAGONS

When Henry cut two similar irregular hexagons out of a sheet of paper, he was surprised to find that he could not completely cover the smaller one with the larger. What were their shapes?

Q-47 WHAT THE DRAFTSMAN MISSED

A blueprint often shows three different views of an object: the front elevation, which is the view from the front; the side elevation, the view from the side at right angles to the front elevation; and the plan view, "looking down" at right angles to the other two views.

Front Elevation



Plan

Here we have shown the front elevation and plan view of a certain object, but without the side elevation. Can you draw the missing side elevation and identify what this object could possibly be?

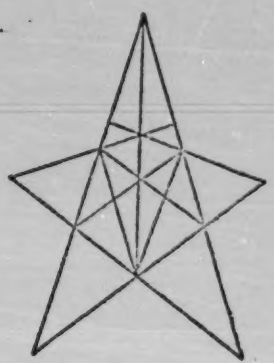
Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 126 & 132

Friedland, Aaron J. Puzzles in Math & Logic, p. 4

Hunter, J. A. H. Hunter's Math Brain Teasers, p. 47

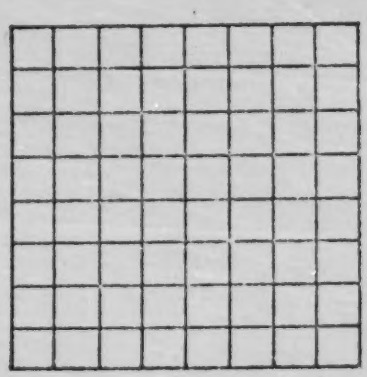
Q-135 CARRY ON COUNTING

Can you count up the number of triangles in this diagram--always remembering, of course, that within one large triangle, there are many small ones. Just one clue: Your answer will be more than 50!



Q-20 CHESSBOARD SQUARES

How many squares, of any size, are made by the lines of a standard chessboard?



Doubleday, Eric. Test Your Wits, Vol. III, p. 135

Longley-Cook, L. H. Work This One Out, p. 20

R-145 A rectangular box has 8 corners. How many edges has it?

R-145 What is the greatest possible number of 1-inch cubes that can be placed into an empty box 4 inches wide by 4 inches deep by 4 inches long?

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, p. 145

S-123 SAM LOYD'S DAISY PUZZLE

An elegant application of the principle of symmetry was propounded by Sam Loyd in his Daisy Puzzle. As he tells the story, he was one day touring in the Swiss Alps and came upon "a little peasant girl gathering daisies. Thinking to amuse the child, I showed her how to prognosticate her matrimonial future, by plucking off the petals of the flower. She said the sport was well known to the country lassies, with the slight difference that a player was always at liberty to pluck a single petal or any two contiguous ones, so that the game would continue by singles or doubles until the victorious one took the last leaf and left the 'stump' called the 'old maid' with your opponent.

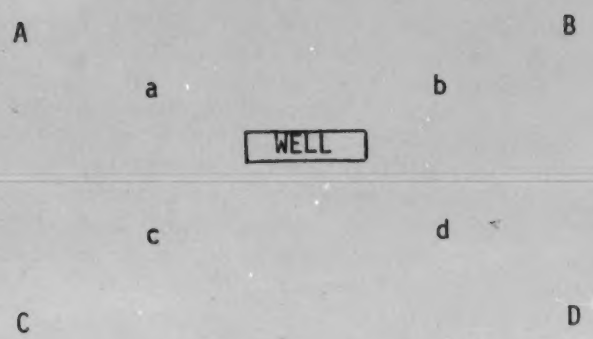
"To our intense astonishment the pretty madchen, who could not have been more than ten years old, vanquished our entire party by winning every game, no matter who played first."

Naturally, the maid must have depended, in one case or the other, upon her opponent's ignorance of the game! The question is, which player can force a win, the first or the second? I shall not spoil the reader's enjoyment of this beautiful puzzle by giving the answer here.

The daisy illustrated by Loyd, it must be stated, has thirteen petals.

Mott-Smith, Geoffrey. Mathematical Puzzles, p. 123

S-126




Families A, B, C, and D are very good friends. They draw their water from a well centrally located and equidistant from the four establishments. Four new families, a, b, c, and d moved in between the old families and the well. A, B, C, and D took a dislike to families a, b, c, and d and decided to build a fence which would surround their homes and the well, and leave the newcomers on the outside of the fence. How did they do this?

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, p. 126

"MONEY" AND "TIME"

T-3 THE CARELESS CLERK

A clerk sold 20 feet of wire and measured it out with a yardstick that was 3 inches too short. How much wire did the customer lack?

T-5 A MELON DRAMA

A farmer surrounded his square melon patch with a ditch 10 feet wide by 20 feet deep. He filled this ditch with water and considered the patch well protected against thieves. However, with only two 9½-foot planks, thieves bridged the ditch and stole the melons. How were they able to do this?

T-17 POST MAN

A man having a garden 10 rods square, wishes to know how many posts will be required to enclose his land, if the posts are placed exactly 1 rod apart!

T-83 SHIPS COMING IN

Three ships leave New York for Le Havre, France, on the same day. It takes the first ship 12 days, the second ship 16 days, and the third ship 20 days to make a round trip. How many days will elapse before all three ships again leave New York on the same day, and how many trips will each ship have made in the meantime?

Mira, Julio A. Mathematical Teasers, pp. 3, 5, 17 & 83

T-10 HIGH NOON

If it takes a clock 15 seconds to strike 6 o'clock, how long does it take to strike 12 o'clock?

T-15 ONE AMONG TWELVE

"I'm just a penny short of half a dollar," said Peter, checking his twelve coins on the table.

"That's funny," his sister told him. "I'm exactly the same, and with twelve coins too. But you've got one coin I haven't got."

What coins did each have?

T-11 HANDS OF A CLOCK

How many times do the two hands of a clock point in the same direction between 6:00 A.M. and 6:00 P.M. of a single day?

T-12 MY CHECKING ACCOUNT

My bank has been charging me 25 cents a month plus 10 cents a check for my checking account. The bank now tells me it will charge, in the future, 50 cents a month plus 8 cents a check and this will save me money. How many checks must I draw a month for this to be true?

Longley-Cook, L. H. Work This One Out, p. 10

Hunter, J. A. H. Hunter's Math Brain Teasers, p. 15

Longley-Cook, L. H. Fun With Brain Puzzlers, pp. 11-12

T-9 KIM'S CAMERA

Kim was showing Joe his new camera. he was particularly proud of having persuaded the dealer to let him have it at less than the regular price of $150.

"It was a funny thing," he said, " that I paid an exact number of dollars altogther."

"Nothing very funny about that," observed Joe.

Kim smiled. "But I paid cash," he told his friend, "dollars, quarters, dimes, nickels, and pennies: the same number of each."

That may have seemed funny to Kim, but it might amuse you to figure out what he paid for the camera.

T-10 KEN'S POCKET-MONEY

First thing every Monday morning Ken receives his dollar; that's one thing the boy doesn't forget. By the beginning of last month he had managed to save a bit, but during the month his hoard dwindled away: each week he spent half of what he had had on the Monday after receiving his pocket-money.

When it came to the fourth Monday of the month, Ken had only $2.75 altogether when he counted his money at breakfast after receiving his weekly dollar. It was just too bad nobody had given him even a cent extra during the month; most times he could be sure of an occasional tip from one of his uncles.

How much did he have that first Monday of last month before his mother handed him that dollar?

Hunter, J. A. H. Fun With Figures, pp. 9-10

T-100 John put $50 in the bank. During the month he withdraws as follows:

| | Withdrawal | | Balance (what's left) |
|---|---|---|---|
| Jan. 6 | $20.00 | leaving | $30.00 |
| Jan. 10 | 15.00 | leaving | 15.00 |
| Jan. 19 | 9.00 | leaving | 6.00 |
| Feb. 3 | 6.00 | leaving | 0.00 |
| | $50.00 | | $51.00 |

To check his account, he adds the withdrawal column and gets $50.00, which is correct. He next adds his balance column and to his surprise finds it adds to $51.00. The question is: Where did the extra dollar come from? It isn't interest. What is the trouble?

T-102 Jim lent Carl 25 cents and said he would charge Carl 4 per cent simple interest for the use of his money; he was just kidding. One week later Carl returned the quarter to Jim and Jim said, "Where is my interest?" Carl replied, "There is no interest due when one borrows a quarter for only one week. Figure it out yourself."

Jim found Carl was right. For how long a time would Carl have to keep Jim's quarter at 4 per cent interest for it to earn 1 cent interest? If Carl borrowed 5 cents at 4 per cent interest, how long would it take for the nickel to earn 1 cent interest?

T-111 A man had two clocks; one did not run at all, and the other lost four minutes every day. Which clock is most often correct? Which should he keep? Explain your answer.

T-167 A man bought a watch for $103, including the tax. He paid for it in eight bills, but they were not five twenties and three ones, and there were no one-dollar bills among the eight. How did he pay for the watch?
Clue: Two dollar bills may be used in the answer.

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 100, 102 & 111

Meyer, Jerome S. Fun With Mathematics, p. 167

V-31 A canoe is floating in a swimming pool. Which will raise the level of the water in the pool higher, dropping a penny into the pool or into the canoe? Or does it make any difference?

V-46 LAD AND DAD

This illustration shows a boy and his father. Which one is taller?

V-8 WATERED MILK

Pitcher A contains a gallon of water, and pitcher B contains a gallon of milk. A cup of water is removed from pitcher A and added to pitcher B, which is then well stirred. A cup of the mixture in pitcher B is now added to pitcher A. Is there more or less milk in pitcher A than water in pitcher B?

Litton, Industries, Inc. The Best of Problematical Recreations, Volume I, p. 31

Mira, Julio A. Mathematical Teasers, pp. 46

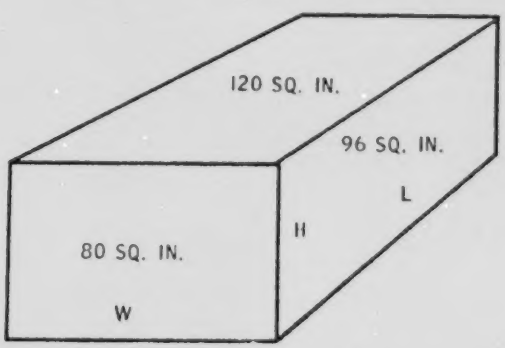
Longley-Cook, L. H. Work This One Out, p. 8

V-108 Tom, Dick, Harry, Joe, and Bill are lined up in these positions, midway through a track meet:

Tom is 20 yards behind Dick.
Dick is 50 yards ahead of Harry.
Harry is 10 yards behind Bill.
Joe is 30 yards ahead of Tom.
Bill is 50 yards behind Joe.

At this point in the race, who is winning? Who is second? third? fourth? last? How many yards ahead is each from the boy next to him?

V-143 The top of a rectangular box has an area of 120 square inches, the side an area of 96 square inches, and the end an area of 80 square inches. What are the dimensions of the box (length, width, and height)?




Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 108 & 143

W-8 WEATHER REPORT

"Can you tell me what the temperature has been at noon for the past five days?" John asked the weatherman.

"I don't exactly recall," replied the weatherman, "But I do remember that the temperature was different each day, and that the product of the temperatures is 12."

Assuming that the temperatures are expressed to the nearest degree, what were the five temperatures?

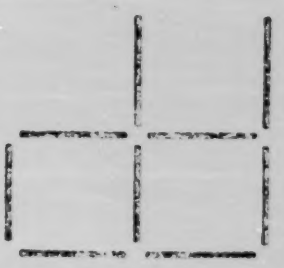
Friedland, Aaron J. Puzzles In Math & Logic, p. 8

X-27 NO MORE STICKS

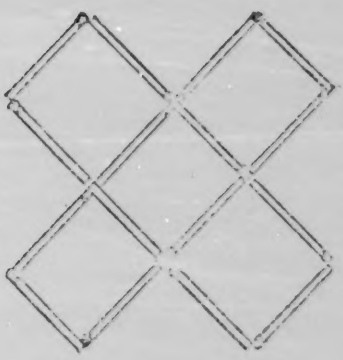
Here are nine matches.

Without breaking or bending, you have to rearrange them to look like three squares.

You have only the nine sticks.



X-73 DIAMONDS GALORE



These five diamonds have been constructed with the use of only 16 matches.

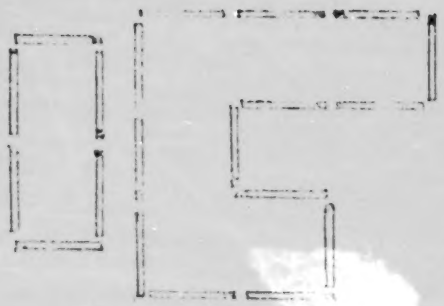
Now with just another eight matches, can you increase the total of diamonds to 14--leaving the existing pattern untouched?

It goes without saying that none of the new matches may be laid across another one.

Hunter, J. A. H. Hunter's Math Brain Teasers, p. 27

Doubleday, Eric. Test Your Wits, Vol. III, p. 73

X-81 MATCH PLAY

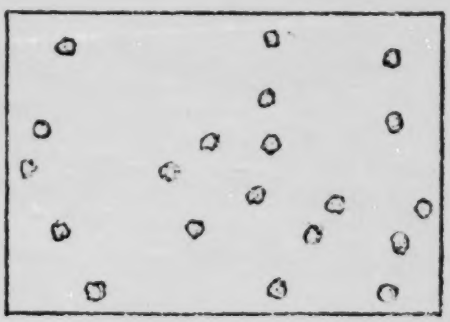


These two diagrams are rather interesting in that the larger is exactly three times greater in area than the smaller.

Now can you take just one match from the larger shape, transfer it to the smaller--and still leave an area three times greater?

You can, of course, rearrange the matches--but no match may be laid on top of another, nor can there be any loose ends.

X-119 BEDS OF ROSES



The park-keeper had 20 prize rose bushes, which visitors to the park would insist on meddling with!

For some curious reason, he decided to fence them off, giving each bush an enclosure of its own. But he had only straight pieces of fencing.

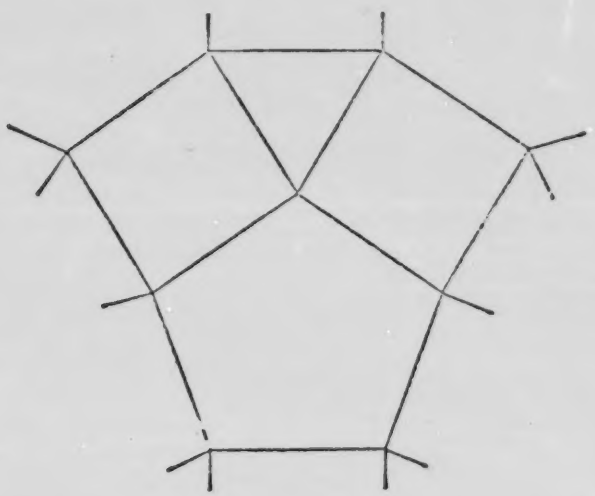
He discovered, eventually, that he needed only six straight fences--so can you draw in six straight lines on the diagram to show how he managed it?

Doubleday, Eric. Test Your Wits, Vol. III, pp. 81 & 119

X-64 THE RHOMBICOSIDODECAHEDRON

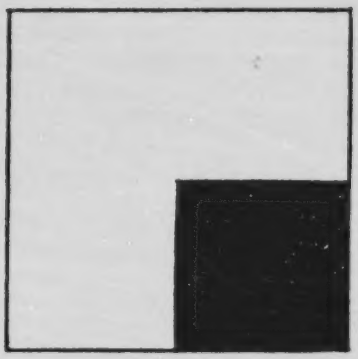
"Do you realize," said Old Prime, "that mathematicians use at times longer and more frightening words than doctors? I bet you do not know what a Rhomb-icosi-dodeca-hedron is. Well, it is one of the Archimedian solids; you might almost call it a sphere with knobs on! It is a symmetrical solid body with plane faces, each of which is a regular polygon (an equilateral triangle, a square, and so on) When looked at from any vertex you see the same arrangement of faces. A cube has 8 vertices (corners) and looked at from any one of these you see three square faces meeting at their corners. With a Rhombicosidodecahedron, four faces meet at each vertex--a square, a pentagon, another square and a triangle. The two squares which meet at each vertex are separated by a pentagon on one side and a triangle on the other. This information defines the solid completely. But to help you visualize it, I will add that each pentagon face is surrounded by 5 square faces, each triangular face is surrounded by 3 square faces and each square face is surrounded by two pentagon faces and two triangular faces alternately. How many vertices and how many faces has a Rhombicosidodecahedron?"

Here is an illustration of a part of the surface of a Rhombicosidodecahedron.



Longley-Cook, L. H. Work This One Out, p. 64-65

X-91 SQUARE DEAL



Farmer Brown's land, as you can see from the diagram, was conveniently shaped like a square.

When he died, he stipulated that a quarter of it--shown by the shaded section--was to go to his widow, while the rest was to be divided equally among his four sons.

Each son ended up with a piece of land identical in shape. Can you show how they solved the problem?

Remember that each son had 3/4 the amount of land that their Mom had.

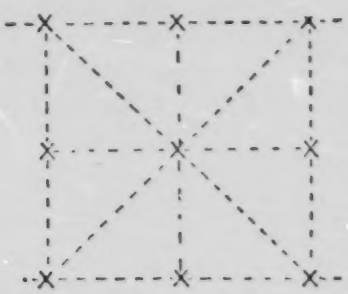
Doubleday, Eric. Test Your Wits, Vol. III, p. 91

X-170 SQUARING THE SQUARE

Transform the square shown below into five small equal squares whose total area shall be equal to this square.

X-17 THE LITTLE ORCHARD

Old Prime had decided to lay out an orchard in his small garden. He had bought 9 fruit trees and I had expected he would plant four at the corners of a square and the other five at the mid points of the sides and diagonals of the square, thus:



"No," said Old Prime, "in that way there are only 8 lines of three trees. I am sure I can do better than that."

How did he plant them to obtain the maximum number of lines containing 3 trees?

Meyer, Jerome S. Puzzle Quiz and Stunt Fun, p. 170

Longley-Cook, L. H. Work This One Out, p. 17

## PROBABILITY AND STATISTICS

Z-90 In a certain high school, 25 per cent of the girls and 50 per cent of the boys attended a football game. If 48 per cent of all the students are girls, what per cent of all the students went to the game?

Z-71 SLOPPY JOE

Sloppy Joe never puts away his socks in pairs, but simply throws them into a drawer. There are fourteen black socks and eight blue socks in the drawer. One night, he needed just one pair of socks the same color, but, in order not to wake up his roommate, Joe did not put the light on; therefore, he had to get the socks in complete darkness. How many socks did Joe have to draw to be sure that he had a pair of matching socks?

Z-105 THE NECKLACE

Eloise has a quantity of glass beads in four colors, red, yellow, green, and blue. She amuses herself by stringing them on wool yarn in various designs. One of her favorite designs is a necklace of 20 beads, in blocks of 4 of a color.

How many different patterns can Eloise make on this plan? She must use the beads in blocks of 4 of the same color, but we will not insist that she make adjacent blocks of a different color. She may make the entire necklace of one color if she chooses. Or she may use two or three or four colors.

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 90

Mira, Julio A. Mathematical Teasers, p. 71

Mott-Smith, Geoffrey. Mathematical Puzzles, p. 105

Z-45 3 TEAMS

Three hockey teams A, B and C play against each other. The following table gives some information about the results.

| | Played | Won | Lost | Tied | Goals for | Goals against |
|---|---|---|---|---|---|---|
| A | 2 | 2 | | | | 1 |
| B | 2 | | | 1 | 2 | 4 |
| C | 2 | | | | 3 | 7 |

Complete the table and find the score in each game.

Z-47 D WAS DUMB

4 teams A, B, C and D are all to play one game of hockey against each other.
After some of the games had been played I managed to get particulars from the secretaries of A, B and C of the numbers of games which their teams had played, won, lost etc. But the secretary of D for some reason firmly refused to give any information at all.

The figures given were as follows:

| | Played | Won | Lost | Tied | Goals for | Goals against |
|---|---|---|---|---|---|---|
| A | 3 | 2 | 0 | 1 | 2 | 0 |
| B | 2 | 1 | 0 | 1 | 4 | 3 |
| C | 2 | 0 | 2 | 0 | 3 | 6 |
| D | | | | | | |

Find the score in each game.

Emmet, E. R. Puzzles for Pleasure, pp 45 & 47

Z-113 PARLIAMENT SOLITAIRE

Here is an easy question concerning the patience game Parliament (also called Tournament).

Two 52-card decks are shuffled together and 8 cards are dealt face up. If not a single ace or king appears in this layout, the game very probably cannot be won, so that it is quite proper to pick up the cards, reshuffle, and try again. That is all you need to know about Parliament to answer the question:

What are the odds that you will turn up at least one ace or king in the first 8 cards?

As the factorials involved are rather large, you need not expand them. Just show the formula and make an estimate of the result.

Z-4 TOSSING COINS

(a) Abner, Boris, and Charlie toss a coin, 15, 16, and 17 times respectively. Which one is least likely to have tossed more heads than tails?

(b) The same, except that the coins are tossed 15, 17, and 20 times, respectively.

(c) The same, except that the coins are tossed 18, 19, and 20 times, respectively.

Z-44 An elimination golf tournament is held for 53 players. How many golf matches will transpire?

Z-49 HEADS AND TAILS

Given four coins, what is the probability that exactly two heads will turn up when the coins are all tossed?

Mott-Smith, Geoffrey. Mathematical Puzzles, p. 113

Friedland, Aaron J. Puzzles in Math & Logic, p. 4

Litton Industries, Inc. The Best of Problematical Recreations, Volume I, p. 44

Longley-Cook, L. H. Fun With Brain Puzzlers, p. 49

AA-86 A bottle and a cork cost $1.10. The bottle cost $1.00 more than the cork. How much does the cork cost? How much does the bottle cost?

AA-86 How can you make five 5's equal 6, using decimals and + and - signs?

AA-88 Jack spent 25 cents for 25 articles of drawing supplies. He bought four different articles:

Paper at 2 sheets for 1 cent.
Pens at 1 cent each.
Pencils at 2 for 5 cents.
Erasers at 5 cents.

How many of each did he buy?

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, pp. 86 & 89

AC-4 MR. YELLOW

You may recall that, on Citrus Island, three tribes--the Whites, the Oranges, and the Lemons--have different standards of truthfulness. (They are otherwise indistinguishable.) Whites always tell the truth; Oranges always lie; Lemon, when asked a series of questions, tell the truth and lie alternately. A Lemon's first answer in a series may either be true or otherwise.

A visitor was somewhat confused recently when he was introduced to three natives named White, Orange, and Lemon, who are--not necessarily respectively--a White, an Orange, and a Lemon. There was also a fourth native named Yellow. The visitors asked each of the first three natives (a) what his own tribe was, and (b) what was Mr. Yellow's tribe.

To these questions, Mr. White replied: "I'm not a White. Mr. Yellow is an Orange."

Mr. Orange replied: "I'm not Orange. Mr. Yellow is a Lemon."

Mr. Lemon replied: "I'm not Lemon. Mr. Yellow is White."

To which tribe does Mr. Yellow actually belong?

(Clue: Concentrate on each ones truthfulness.)

AC-18 BATTLE OF WITS

John and Mary were playing at Patball, an outdoor game of their own invention. In skill, there was nothing to choose between them.

They were playing to 21 points.

Mary had the sun in her face. "Look here, John," she said, "with the sun where it is now you're scoring about three points to my one. I ought to have a handicap."

"Okay," said John. "Or why not change over at half time? Say, when I've scored 10 points."

"Right," said Mary. She considered. "After all, John, you did win the toss. Let's change over when you've scored 12."

"A generous offer," said John. "I'll accept it, all the same."

What should be the result of the game?

Phillips, Hubert. My Best Puzzles in Logic and Reasoning, pp. 4 & 13

AC-23 TWENTY MARBLES

A bag contains 20 marbles. They are of three different colors. There are 8 blue ones, 7 red ones, and 5 green ones.

You are invited to close your eyes and draw the maximum number of marbles consistent with your leaving in the bag (1) at least four marbles of any one color, and (2) at least three marbles of any second color.

How many should you draw?

AC-33 EQUESTRIAN

Catkin, Jorkin, Lambkin, and Pipkin each named his favorite horse after one of the other three. In a recent trial, each rode one of the four horses. None, however, rode either his own horse or the horse named after him.

Lambkin rode Catkin's horse. The horse owned by Pipkin was ridden by the namesake of Lambkin's horse. The owner of Catkin rode Pipkin.

Can you name the horse of which the rider of Lambkin is the owner?

AC-35 "I'M A WASHINGTON"

You may recall that in the village of Tellham Nuthen everyone's surname is either Washington or Longbow. The Washingtons always tell the truth, the Longbows always tell lies.

Meeting a group of four villagers in the street, I put my customary inquiries as to whether they were Longbows or Washingtons. The first villager replied: "We are all Longbows." The second: "Only one of us is a Longbow." The third: "No; two of us are Longbows." The fourth: "I'm a Washington."

Now, a villager who says "I'm a Washington" may clearly be either a Washington or a Longbow.

Was the fourth speaker a Washington?

AC-175 Three men, traveling with their wives, came to a river which they had to cross. There was but one boat and only two could cross at one time. Since the husbands were jealous, no woman would be left with a man unless her husband was present. How did they get across that river?

Phillips, Hubert. My Best Puzzles in Logic and Reasoning, pp. 23, 33 & 35

Frohlichstein, Jack. Mathematical Fun, Games and Puzzles, p. 175

AC-178 THE WISE TOURIST

Before the great flood, there existed a land inhabited by two tribes, the Alphas and the Omegas. The Alphas were such liars that they never told the truth and the Omegas were so honest that they always told the truth. One day, a tourist arrived from a distant land, and he wanted a guide to take him around and show him the country. Knowing the reputation of the inhabitants, he wanted to hire an Omega as a guide. He approached a group of three men and asked the first whether he was an Alpha or an Omega. This man, who spoke a dialect, said something to the tourist which he could not understand. The second man then said to the tourist, "He said that he is an Omega. He is an Omega, and so am I." However, the third man promptly stated, "That's a lie," and pointing to the first man, he said, "He is an Alpha and I am an Omega." The tourist without hesitation hired one of the men, who was an Omega. How did he know which one of the men to hire?

AC-179 THE QUICK-THINKING EXECUTIVE

Joe, Jack, and Jim are executives of a large corporation. All three of these men are very intelligent. The president of the corporation wanted to advance one of them to executive vice-president; so he decided to give the job to the one who showed the greatest ability for quick, logical thinking. To find the quickest thinker among them, he had all three men seated in a room without mirrors so that they were facing each other. Then he said to them, "I am going to put the lights out; then I shall mark each of your foreheads with either a black or a white spot. After doing this, I shall turn the lights back on, and if any one of you sees a black mark on the forehead of one of the others, stand and remain standing until one of you can let me know the color of the mark on your own forehead. The first one who can do this will be appointed executive vice-president.

Accordingly, the president turned the lights out and marked each man's forehead. Then he turned the lights on again and all three of them stood up. Immediately, Joe said, "I have a black mark." How did he know this?

Mira, Julio A. Mathematical Teasers, pp. 178, 179

AC-181 THE BIRTHDAY PARTY

At a birthday party for one of my grandchildren, Tony and Chris wanted to sit next to Cookie. For the sake of peace and order, Paul should not sit next to Tony or Chris. Peggy does not want to sit next to Tony or Chris. Tory, whose actual name is Victoria, and Jeffer will sit opposite each other. Bill wants to sit next to Cathy, but Cathy wants to sit next to Jeffer. Finally, Julia does not want to sit next to Bill. While these arrangements are being discussed, Paul and Cathy are already seated and are beginning to help themselves to the ice cream and cake. I want to arrange the seating so that the boys and girls are alternately seated. Luckily, the seats taken by Paul and Cathy, on the round table, allow me to seat them all in a satisfactory arrangement. How were they seated?

AC-189 UNTIE THIS NUPTIAL KNOT

Dan, Pete, and Sid are married to Cora, Jill, and Flo, but not necessarily in this order, and they are all members of the same golf club. In the club's golf tournament,

(1) Dan's wife and Jill's husband play golf against Cora and Flo's husband.
(2) No husband and wife play as partners.
(3) Pete, who is a tennis player, does not play golf.

What is the name of each man's wife?

AC-188 WHO'S WHO?

Clark, Morris, and Howe are a physicist, a pianist, and a philosopher, but not necessarily in that order. Determine who's who by the following facts:

(1) Morris makes more money than Clark.
(2) Clark has never heard of Morris.
(3) The philosopher tried to get the pianist to do a benefit for his charity, but the pianist refused, because he was doing some experiments on sound with the physicist.
(4) The physicist makes more money than the philosopher.

Mira, Julio A. Mathematical Teasers, pp. 181, 188 & 189

AC-189 THE BRIDGE GAME

Bill, Sam, and Jack are married to Grace, Joan, and Becky, but not necessarily in that order. Last Friday, they played bridge as usual, but no wife was a partner of her husband. In one of the rubbers,

(1) Bill and Grace play Jack and Becky.
(2) Joan plays with Jack as a partner.
(3) Bill plays with Sam's wife as a partner.

What is the name of each man's wife?

AC-189 WHO VERIFIES THE ACCOUNTS?

Cronin, Baker, and Ennis are the controller, manager, and treasurer of a corporation, but not necessarily in that order.

(1) The treasurer, who is single, earns the least.
(2) Ennis is a brother-in-law of Baker.
(3) Ennis earns more money than the manager.

Who is the controller?

AC-8 THE TRUTH WILL OUT

One experience when over in Kalota last fall really had me puzzled.

You know that odd custom in the island? Men always tell the truth. A woman, however, never makes two consecutive true or untrue statements: if one is true, her next is a lie, and vice versa. And the same goes for the boys and the girls.

Meeting a Kalotan couple with their child, I asked the kid. "Are you a boy?"

The reply was in Kalotan!

Fortunately, the parents both spoke our language.

"Kibi said, 'I am a boy'," one of them declared.

"Kibi is a girl, Kibi lied," the other parent told me.

Well! What can you make of that?

Mira, Julio A. Mathematical Teasers, pp. 189

Hunter, J. A. H. Hunter's Math Brain Teasers, p. 8

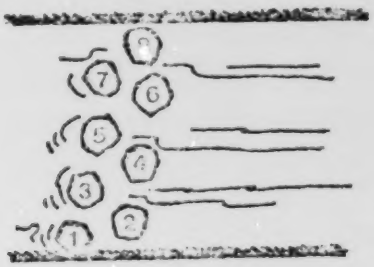
AC-12 THE BLACK AND WHITE FOREHEADS

There are five men, A, B, C, D, E, each of whom has a white or a black disc attached to his forehead. Each man can see the discs worn by the other four, but is unable to see his own. If a man is wearing a white disc, any statement he makes is true; if he is wearing a black disc, any statement he makes is false. Statements are made as follows:

A says: "I see three white discs and one black."
B says: "I see four black discs."
C says: "I see one white disc and three black discs."
E says: "I see four white discs."

Deduce the colors of the discs worn by all five men.

AC-21 QUICK STEP




There are eight stepping stones across this river--but you must obey a few simple rules to reach the opposite bank.

You must return once to the near bank, and land twice on the far bank (the second time, you can stay there!

On the way, you must step on each stone the same number of times. How many steps will you take altogether?

Emmet, E. R. Puzzles For Pleasure, p. 12

Doubleday, Eric. Test Your Wits, p. 21

AC-45 ROWING BLUES

It was a lovely day--just right, thought the Smith family, to have a picnic on the island.

But the notice on the boat clearly warned them that it was capable of carrying only 160 pounds at one time--which posed a problem, since Mr. and Mrs. Smith each weighed exactly 160 pounds, and their two sons each weighed just 80 pounds.

Nor did they want to leave the picnic basket behind and that weighed another 25 pounds.

Happily, the four senior members of the family could all row--so how did they cross to the island in the fewest possible journeys?

AC-9 CAFE AU LAIT

With two identical cups of hot coffee, which will cool to drinking temperature first: the one to which cream is added immediately or the one to which the same quantity of cream is added later?

AC-19 THE THIRSTY HUNTERS

We can't reveal the full story of this shocking incident, but the main facts can be recorded.

Two hunters, still thirsty after a long day in the woods, wanted to share their last bottle of rye. They contrived to divide the twenty-four ounces of liquor exactly equally, and with only seven distinct transfers from container to container, although they had only that bottle and two other containers, an eleven-ounce and a five-ounce measure.

How did they manage to do it?

AC-31 ALL IN A FAMILY

"Relationships are funny," remarked Bob. "Tom's the same relation to you that I am to your son."

"So he is," Charlie chuckled. "And you're the same relation to me that Tom is to you."

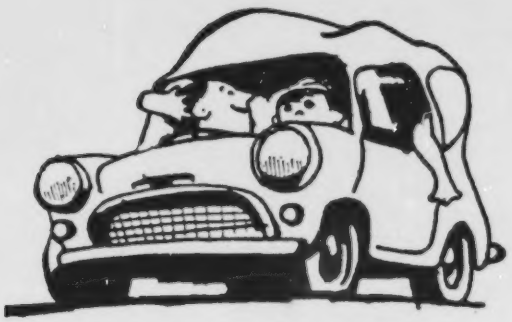
What relation was Charlie to Tom?

Doubleday, Eric. Test Your Wits, p. 45

Longley-Cook, L. H. Work This One Out, p. 9

Hunter, J. A. H. Hunter's Math Brain Teasers, pp. 19, 31

AC-43 FAMILY TREE



It was a bit of a squash getting all this family into the car for a Sunday drive.

But, as one of the men pointed out, it wasn't quite as bad as it sounded!

You see, there was a grandfather and a grandmother; a father-in-law, a mother-in-law and a daughter-in-law; two daughters, two sisters and two sons; a brother, two fathers and two mothers--plus four children and three grandchildren!

Actually, there were fewer than ten of them in the car. But how many exactly?

AC-141 SPOT THE LIAR

There was a tremendous crash from the dining-room. When Jane rushed in, she found her four sons standing around the remnants of her prize Crown Derby tea set.

"Harry did it," said Tom. "Well, it certainly wasn't me," added Dick.

"If Tom says it was me, he's lying," countered Harry.

George tucked his chin into his chest, and muttered sullenly: "It was Tom."

Assuming one boy was lying, and the other three were telling the truth, who was the guilty party?

Doubleday, Eric. Test Your Wits, pp. 43 & 141

AC-192 WHOSE LITTLE GIRL ARE YOU?

During the father-daughter weekend at a women's college, six couples took part in the table tennis competition. The fathers' names were William, Robert, Thomas, Edward, Arthur, and Carl; the daughters' names were Pat, Joan, Dorothy, Nancy, Mary, and Helen, but not necessarily in that order. Their homes were in New Jersey, Virginia, Connecticut, Pennsylvania, Massachusetts, and New York.

1) Robert and Edward played doubles against Joan and Mary.
2) The girl from Pennsylvania played singles against Dorothy, Pat, Helen, and Thomas.
3) William and Carl played doubles against Pat and Mary.
4) The girl from Connecticut played singles against Mary, Dorothy, Pat, and Carl.
5) Mary and Joan played doubles against Robert and William, and then against Carl and Edward.
6) Dorothy played singles against Nancy and Helen.
7) The girl from Virginia played singles against Dorothy.
8) Joan played singles against William, Carl, and Thomas.
9) The girl from New Jersey played singles against Robert and William.
10) Edward played singles against Pat and Nancy.
11) Pat played singles against Dorothy, Helen, and Nancy.
12) William played singles against Helen and Nancy.
13) Nancy played singles against the girls from Pennsylvania and New Jersey.
14) Nancy played singles against Pat and Helen.
15) The girl from Massachusetts played singles against Dorothy and Nancy.

No father and daughter ever took part in the same match. Find the father-daughter relationship and their respective home states.

AC-6 THE TEMPLE OF TORPOR

If a man is over forty he cannot be a member of Yonkers Youngsters. Everyone of importance in Salem is either a member of Yonkers Youngsters or of Iowa Idlers or both. Only if one is allowed to enter the Taunton Temple of Torpor can one be a member of Iowa Idlers. You cannot both not be under thirty and be allowed to enter the Taunton Temple of Torpor.

Smith is someone of importance in Salem and is not under thirty. What else can you say about him?

Mira, Julio A. Mathematical Teasers, p. 192

Emmet, E. R. Puzzles For Pleasure, p. 6

APPENDIX B

ANSWER KEY

A-56 TOM'S CLASS

This type of problem is best solved by use of a diagram. If we draw three overlapping circles representing the three subjects and indicate by numbers inside the circles the children who like each subject, the problem can be quickly solved. (See Figure 27) We can place 3 in the area covered by all three circles to represent the three children who liked all three subjects.

Now 16 said they liked English and Languages, and these must all be included in the shaded area consisting of the overlapping of the English and Languages circles. The 3 already indicated in the top half like all three subjects so the balance of 13 must be placed in the bottom half. These are the children who like English and Languages but do not like Mathematics. Proceeding in this manner, using the information that 7 liked both Mathematics and Languages and 25 in total liked Languages we have the representation shown in Figure 28. Proceeding similarly with the other information, we can complete the whole diagram. (See Figure 29) Counting the total number of children in each area of the diagram, we find there are 47 children in Tom's class.

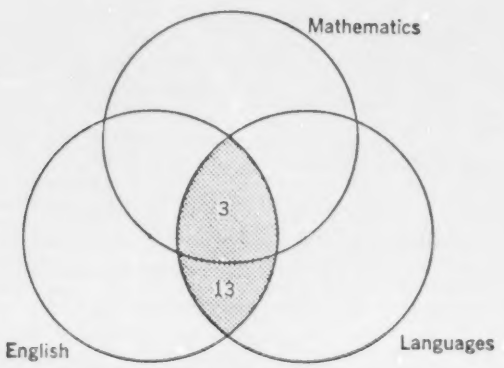



FIGURE 27

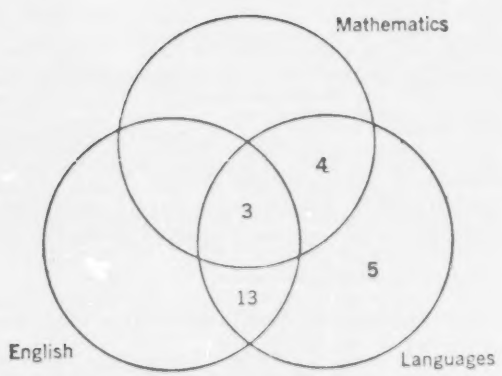



FIGURE 28

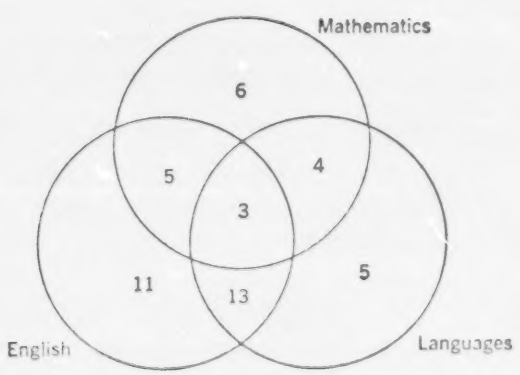



FIGURE 29

B-56 N, if we consider the letters as the first letters in one, two, three, four, five, six, seven, eight, nine.

*** ADDITION ***

D-111 Put number 1 on any plate and then, moving clockwise, in the plates with 2, 3, 4, 5, 6, 7, 8 and 9. In the plate to the left of 1, go anti-clockwise with 11, 12, 13, 14, 15, 16, 17, 18, 19. Keep number 10 for the center.

D-127 6 - 19 - 25

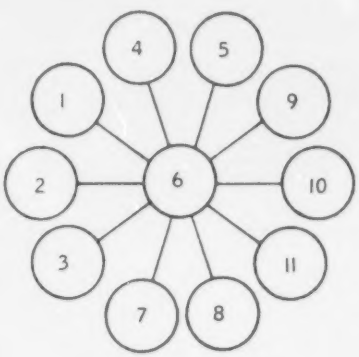
D-10 Here are three possible solutions:

| | | |
|---|---|---|
| 6 | 1 | 8 |
| 7 | 5 | 3 |
| 2 | 9 | 4 |

| | | |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |

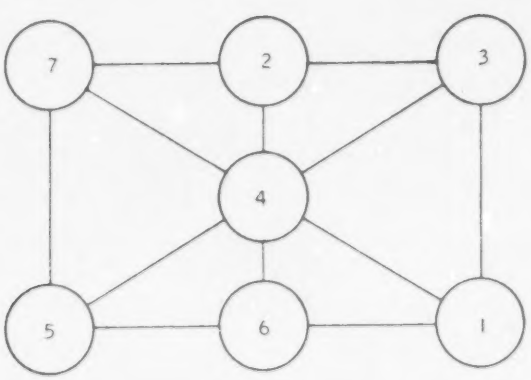
| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

D-10 The sum, in all directions, is 18.




D-11 When you put them all together, this is only one haystack.

D-11




(adds to 12 in all directions)

D-12 The clue to the solution of this is the fact that in the "magic square" with "magic constant" 15 the central cell contains 5: one third of 15.
Then, since 111 = 37 X 3, we can write 37 in the central cell of our semicompleted square. The remaining cells can then be filled in to give the required "magic constant" 111.

| | | |
|---|---|---|
| 31 | 73 | 7 |
| 13 | 37 | 61 |
| 67 | 1 | 43 |

An interesting point about this "magic square" is that all nine numbers are prime numbers.

D-41 A solution is 123 + 4 - 5 + 67 - 89 = 100.

D-166 8 + 8 + 8 + 88 + 888 = 1000

D-127 Suppose you are 19 and the other person is <u>older</u>. Add the result 2 to your age 19; thus, 19 + 2 = 21, which is his age. If the other person is <u>younger</u> and you are 23, subtract his result 4 from your age 23; thus, 23 - 4 = 19, which is his age.

<u>Explanation</u>. Let "a" denote your age and "b" represent the other person's age. If he is <u>older</u> than you, the operations yield 99 - a + b - 100 + 1 = -a + b = b - a; that is, the result b - a is the difference in your ages. Hence, if you add b - a to your age, you will obtain b - a + a = b which is his age.

If the other person is <u>younger</u>, the operations yield 99 + a - b - 100 + 1 = a - b; that is, the result a - b is the difference in your ages. Hence, if you subtract a - b from your age, you will obtain a - (a - b) = a - a + b = b, which is his age.

D-133 You throw 3 and 5 forming the number 35 (or 53). Suppose you select 3; then the successive operations yield [(3 X 5) + 7] X 2 + 5 = 49. Subtract 14 from this final sum 49; that is 49 - 14 = 35, to obtain the original throw 3 and 5. If you had selected 5, the final sum is [(5 X 5) + 7] X 2 + 3 = 67; subtract 67 - 14 to obtain 53, which is the original throw 5 and 3.

D-12

| | | |
|---|---|---|
| 20 | 1 | 12 |
| 3 | 11 | 19 |
| 10 | 21 | 2 |

(adds to 33 in all directions)

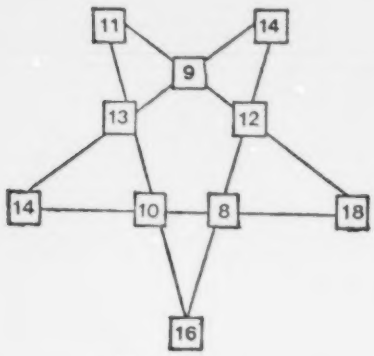
D-15 Here are three solutions:

| | | | |
|---|---|---|---|
| 1 | 15 | 14 | 4 |
| 12 | 6 | 7 | 9 |
| 8 | 10 | 11 | 5 |
| 13 | 3 | 2 | 16 |

| | | | |
|---|---|---|---|
| 1 | 14 | 15 | 4 |
| 8 | 11 | 10 | 5 |
| 12 | 7 | 6 | 9 |
| 13 | 2 | 3 | 16 |

| | | | |
|---|---|---|---|
| 1 | 12 | 13 | 8 |
| 15 | 6 | 3 | 10 |
| 4 | 9 | 16 | 5 |
| 14 | 7 | 2 | 11 |

D-101




*** SUBTRACTION ***

E-27 (a) The catch in this problem is "three hundred and WON." The answer is 536, not 537. Most people think you mean "one" (1).

| Correct | Incorrect |
|---|---|
| 236 | 236 |
| 300 | 301 |
| 536 | 537 |

(b) 19. There were twenty sick sheep, not twenty-six sheep.

E-121 Divide the difference 91 - 19 = 72 by 9 obtaining 8. This number 8 is the difference between the two digits 9 and 1.

Explanation. Let "h" represent the hundred's digit and "u", the unit's digit.

The number is $10h + u$
The new number is $10u + h$
The difference is $9h - 9u = 9(h - u)$
Thus, if the difference between the two numbers is denoted by "d", then

$$9(h - u) = d \text{ and } h - u = \frac{d}{9}$$

That is, the difference between the two digits h - u is equal to the difference between the two numbers "d" divided by 9.

E-121 7654 - 4567 = 3087. No matter what number of four digits you write such that the digits decrease by one from left to right, the difference between this number and the number formed by reversing the digits is always 3087.

Explanation. If "a" represents the thousand's digit, then a - 1 is the hundred's digit, a - 2 is the ten's digit and a - 3 is the unit's digit. The number is

$$1000a + 100(a - 1) + 10(a - 2) + (a - 3)$$

Reversing the digits, the new number is,

$$1000(a - 3) + 100(a - 2) + 10(a - 1) + a$$

The difference is

$$\begin{array}{l} 1000a + 100(a - 1) + 10(a - 2) + a - 3 \\ \underline{1000(a - 3) + 100(a - 2) + 10(a - 1) + a} \\ 3000 + 100 + (-10) + (-3) \\ \qquad\qquad\qquad\qquad\qquad\qquad + 3087 \end{array}$$

That is, no matter what number of four digits you write, such that the digits decrease by one from left to right, the difference between this number and the number formed by reversing the digits is always 3087.

E-123 765 - 567 + 198. No matter what number of three digits you write, where the digits decrease by one from left to right, the difference between this number and the number formed by reversing the digits is always 198.

Explanation. If "a" represents the hundred's digit, then a - 1 is the ten's digit and a - 2 the unit's digit. So that the number is

$$100a + 10(a - 1) + (a - 2)$$

Reversing the digits, the new number is,

$$100(a - 2) + 10(a - 1) + a$$

The difference is

$$\begin{array}{l} 100a + 10(a - 1) + (a - 2) \\ \underline{100(a - 2) + 10(a - 1) + a} \\ 200 + 0 + (-2) = 198 \end{array}$$

That is, no matter what number of three digits you write, such that the digits decrease by one from left to right, the difference between this number and the number formed by reversing the digits is always 198.

E-133 The large diamond is worth $3,000.

## *** MULTIPLICATION ***

F-35

```
   573 (multiplicand)
x  219 (multiplier)
------
  5157
  573
 1146
------
125487
```

F-36 There are three possible answers. Two of them are:

```
  123          113
  163          133
 ----         ----
  369          339
 738          339
 123          113
-----        -----
20049        15029
```

F-16 If 6 boys fill 6 notebooks in 6 weeks,
12 boys fill 12 notebooks in 6 weeks,
and 12 boys fill 24 notebooks in 12 weeks;
If 4 girls fill 4 notebooks in 4 weeks,
12 girls fill 12 notebooks in 4 weeks,
and 12 girls fill 36 notebooks in 12 weeks;
Hence 12 girls and 12 boys fill 24 + 36 = 60 notebooks in 12 weeks.

F-7 $105.00. There are 11 posts.

F-125 The difference between the squares of 11 and 12 is 144 - 121 = 23. Subtract 1 from this difference and divide by 2. The result (23 - 1) ÷ 2 = 11, which is the number you selected.

F-124 No matter what number of four digits you choose such that the digits decrease from left to right, the result of these operations is always 10890.

F-19 Believe it or not, the pile of paper will be over 111 miles high!

## ***DIVISION***

G-115 The sum of the digits of 765932 is 7 + 6 + 5 + 9 + 3 + 2 = 32. Find the difference between 32 and the nearest integral multiple of 3 which is greater than 32. Thus, the nearest integral multiple of 3 greater than 32 is 33 and the difference, 33 - 32 = 1. The digit 1 can be placed before, after, or anywhere within the number 765932 and the resulting number is divisible by 3. Thus, 1765932, 7651932, 7659321, etc. are divisible by 3.

G-115 The sum of the digits 765932 is 7 + 6 + 5 + 9 + 3 + 2 = 32. Find the difference between 32 and the nearest integral multiple of 9, which is greater than 32. Thus, the nearest integral multiple of 9 greater than 32 is 36. The difference, 36 - 32 = 4, hence a 4 can be placed before, after, or anywhere within the number 765932 and the resulting number is divisible by 9. Thus, 4765932, 7659324, 7654932, etc. are divisible by 9.

G-116 Any number divided by 11 leaves the same remainder as the difference between the sum of its digits in the odd places and those in the even places divided by 11. It then follows that a number is divisible by 11 if the difference between the sum of its digits in the odd places and those in the even places is either divisible by 11 or equal to 0.

G-120 The number 134 divided by 9 gives a remainder equal to 1 + 3 + 4 = 8. The product 134 x 7 = 938 and if we divide this product 938 by 9, the remainder is 9 + 3 + 8 = 20, or 2 + 0 = 2. The product 8 x 7 = 56 and the remainder when 56 is divided by 9 is 5 + 6 = 11 and 1 + 1 = 2.

$$N \times d = d(9q + r) = 9dq + (dr).$$

Thus, the remainder upon dividing the product N x d by 9 is the product "dr", which is the product of the first remainder by the number we chose for the multiplier.

G-58 9 minutes--the tenth piece will require no cutting.

G-60 1 + 2 + 3 + 4 + 5 + 6 - 7 + 8 = 9. (Note: This is the answer as it appears in the original source. Observe that the author has not used the 4 processes but only 3.)

***FRACTIONS***

H-78 The father died at 84, the son at 42 when his father was 80. The father's life: childhood, 14 years; youth, 7 years; bachelorhood, 12 years; married at 33; the son was born when his father was 38. This problem may be worked by addition of fractions:

$$\begin{aligned} 1/6 &= 14/84 \\ 1/12 &= 7/84 \\ 1/7 &= 12/84 \\ &\overline{\quad 33/84} \end{aligned}$$

He must have died at 84 since the denominators of the fractions 6, 12, and 7, have 84 as their least common denominator. He was married at 33 because 33/84 of his life came before his marriage. His son came 5 years later when the father was 38, and died at 42, half the age of his father when he died. The father was 80 years old at the time of the problem.

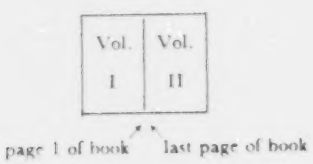
H-79 1½ days.

H-79 There are only 3 women, not 4. They were grandmother, mother and daughter. Two were mothers: grandmother and mother. Two were daughters: mother and daughter. (1/3 of $18 = $6.) They spent $6.

H-81 He had 8 apples. Twice as many would be 16, half as many would be 4; the total is 20.

H-76 If you say 2¼ inches, you are with the majority; however, you are wrong. The worm travelled ¼ inch.

When two volumes stand together on a bookshelf, Volumes I and II respectively, the first page of Volume I and the last page of Volume II are separated only by two covers. Try it yourself and you'll believe it. Each cover is 1/8 inch thick; thus two covers are 2/8 or 1/4 inch thick, and the worm ate its way through the two covers or ¼-inch thickness.




H-76 The man simply laid down one of his boards across the corners of the land and, placing the other board on top of that, was able to span the intervening distance and walk safely over to the castle.

H-77 There is a trick to this problem: "½ of them study mathematics, 1/3 of them study geometry..." Geometry is a part of mathematics (included in the general field of mathematics), this 1/3 is included in the ½ for mathematics. There are really only two fractions used --one for mathematics and one for chemistry.

H-88 6 days. Since each day the students will do twice the number of problems already done, when one third of the problems are done at the end of 5 days, the students will do two thirds of the problems the next day; hence, they will complete the assignment in one more day. Therefore, it will take them 6 days.

## *** NUMBER THEORY AND ORDER ***

J-14 No solution

J-45 The missing number is 31. The terms of the sequence are the representations of 16 to various bases starting with base 16 and proceeding consecutively to base 2.

## *** INTEGERS ***

K-11 There cannot be more than 50 integers in the group or there will be some pair totaling 100. There are a number of solutions of 50 integers, of which the simplest is the group of integers 50 through 99.

K-50 Each term consists of the first digit of the preceding term multiplied by the second digit of the same term. Thus,

$$49 = 7 \times 7$$
$$36 = 4 \times 9$$
$$18 = 3 \times 6$$

Hence, the fifth term is $1 \times 8 = 8$.

K-12 We are told that each letter represents a digit in the following problem:

$$\begin{array}{r} ABA \\ -CA \\ \hline AB \end{array}$$

From this we deduce

$$AB + CA = ABA$$

Now, looking at the letters in the units position.

$$B + A = A$$

This is only possible if B = 0 (zero), and we have

$$A0 + CA = A0A$$

Looking at the letters in the tens and hundreds positions,

$$A + C = A0$$

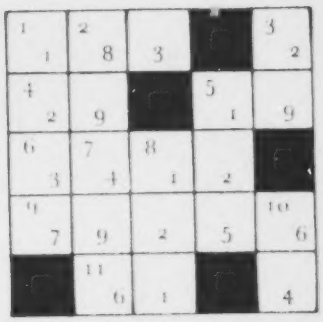
Now A and C cannot be greater than 9, and since they are different, their sum cannot be greater than 17. Hence A0 cannot be greater than 17 and A must be 1.
Therefore, $1 + C = 10$.
Giving C = 9 and the whole solution is

$$\begin{array}{r} 101 \\ -91 \\ \hline 10 \end{array}$$

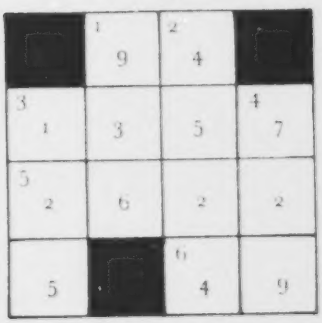
K-77 (a) 36. (b) 27. (c) 25 or 36. (d) 54. (e) 29,38,47,56.

K-16 $\frac{4}{.4} \times \frac{4}{.4} = 10 \times 10 = 100$
also - with four 9's: $99\frac{9}{9} = 100$.

K-61 Complete solution:



K-62 Complete solution:



*** WRITING SENTENCES ***

L-4 Say she had $X before her find. Then (X + 2) = 5(X - 2), so X = 3. Jill had $3.00 before her lucky find.

L-7 One of them had "x" coffees @ 10¢, "y" pieces of pie @ 15¢, and "z" ice creams @ 20¢, where none of x, y, z can be zero.
So 10x + 15y + 20z = 105; hence 2x + 3y + 4z = 21, which has possible solutions:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Coffees | x . . | 1 | 1 | 2 | 3 | 4 | 5 | 7 |
| Pie | y . . | 1 | 5 | 3 | 1 | 3 | 1 | 1 |
| Ices | z . . | 4 | 1 | 2 | 3 | 1 | 2 | 1 |
| Total items . | | 6 | 7 | 7 | 7 | 8 | 8 | 9 |
| Total "eats" | | 5 | 6 | 5 | 4 | 4 | 3 | 2 |

Sam had as many coffees as the total of all items that each of the other two had. Hence, from the tabulation, Sam had 7 coffees, 1 pie, and 1 ice cream.
Then Ted and Joe each had 7 items, Ted's "eats" being more than Joe's. But they all had different numbers of ice creams, so we have: Ted, 2 coffees, 3 pies, and 2 ice creams; and Joe, 3 coffees, 1 pie, and 3 ice creams.

L-28 Say length of head was 2x feet. Then 6x + 3x = 27/2, so 2x = 3. The head was 3 feet long.

L-13 My daughter is 22 years old.

L-15 If Bill is age x, Jim is 30 - x. In 14 years Jim will be 30 - x + 14= 44 - x. This must equal three times Bill's age or 3x.

$$\therefore\ 44 - x = 3x$$
$$\text{or} \quad 4x = 44$$

Bill is 11 and Jim is 19.

L-10 60 miles. He leaves at 9 a.m.

L-3 Working backwards the answer is $12, $21, and $39.

L-28 The problem is wrongly stated. The men did not actually pay $27 for the room, but $25. If the problem is stated this way, the amount will total $30. The cost of the room ($25 + $3 rebate + $2 the bellboy kept) is $30.
Another way to state the problem is that the room costs $30, less a $3 rebate, leaving $27. Now, here is the fallacy of the originally stated problem. The $2 the bellboy got is not added to $27 to get $29, but should be subtracted from the $27 to get $25. The hotel clerk rebated $5 - $3 the men got back, but the $2 was kept by the bellboy. The total is $30.

L-66 Say "x" cutlets, "y" chops. Then 7x + 11y = 94, with x and y whole numbers. Quick trial gives x = 4 and y = 6.
Button ate 4 cutlets and 6 chops.

L-47 From the information about bicycle times and speeds we establish that the ratio of the distance to Monk's Wolton and Bishop's Wolton is 4 to 3. Since the three towns are at the corners of a right-angled triangle whose hypotenuse is 5 miles, the actual distances must be 4 and 3 miles, respectively; therefore, the distance to Monk's Wolton is 4 miles.

L-17 Jones's total was 72.

L-29 80 geese.

*** SOLVING SENTENCES ***

M-25 The horse had seven sacks and the donkey had five.

M-39 Jack is 10, and Jill is four.

M-9 One inch

*** RATIOS ***

N-3 The pole is 28 feet high. The problem is solved by simple proportion. The height of the pole is to its shadow, 21, as the man, 6, is to his shadow, $4\frac{1}{2}$.
The well is a trifle under 22 feet deep. If the drum is 7 inches in diameter its circumference is $7\pi$ and $\pi$ is about 3.1416. A rope that wraps 12 times around it is 12 x 7 x 3.1416 inches long; cancel the factor 12 and you have the answer in feet.

The answer to the problem of the sheep is not 60, but 55 sheep. If it takes 10 sheep 10 minutes to jump over a fence--the time being measured from the jump of the first sheep to the jump of the 10th--then the interval between jumps is 10/9 of a minute. There are 60 + 10/9 or 54 such intervals in an hour, so that 55 sheep jump the fence in this time.

N-15 7 months.

N-15 Jim gave up 1 apple, so received 3¢. Paul gave up 3 apples, so received 9¢.

*** 3-DIMENSIONAL FIGURES ***

P-127 Move number 9 line, and place it on top of lines 1 and 3.
Move number 10 line and place it between lines 5 and 6.

*** 2-DIMENSIONAL FIGURES ***

O-119 Twenty triangles.

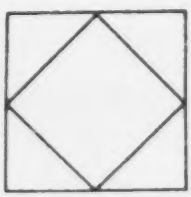
Q-120 Fifty-one rectangles. They are:

| | | |
|---|---|---|
| 1 | 2-4-6 | 5-6-12 |
| 2 | 4-6-8 | 6-12 |
| 3 | 4-6 | 1-2-3-4 |
| 4 | 6-8 | 1-2-3-4-5-6 |
| 5 | 1-3 | 3-4-5-6 |
| 6 | 1-3-5 | 3-4-5-6-7-8 |
| 7 | 3-5 | 5-6-7-8 |
| 8 | 5-7 | 9-10-3-5 |
| 9 | 9-3 | 9-10-3-5-4-6 |
| 10 | 9-3-4 | 3-4-5-6-11-12 |
| 11 | 3-4 | 4-6-11-12 |
| 12 | 3-5-7 | 1-2 |
| 1-2-3-4-5-6-7-8 | 3-4-11 | 7-8 |
| 1-3-5-7 | 4-11 | 9-10 |
| 2-4-6-8 | 10-5 | 11-12 |
| 9-3-4-11 | 10-5-6 | |
| 10-5-6-12 | 5-6 | |
| 9-3-4-11-10-5-6-12 | | |
| 2-4 | | |

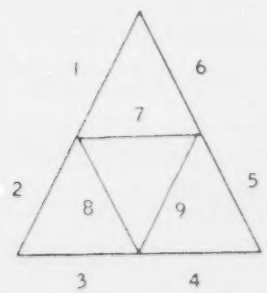
Total: 51

Q-125 One possible solution: Follow the numbers: start at 4-3-2-1-7-10-6-9-8-5

Q-126



Q-132




(9 lines of equal length, as indicated, and 5 equilateral triangles.)

Q-4 Once it is realized that the hexagon need not be convex, it is easy to find any number of solutions similar to the one shown below.

Q-47

*Side Elevation*

This could be a wire figure, in the form of an ellipse, folded at its smaller "diameter."

Q-135 There are 62 triangles.

Q-20 If the small squares have 1-inch sides, there are $8^2 = 64$ squares; $7^2 = 49$ 2-inch squares, etc., down to 1 8-inch square. The total number is 204.

*** SIMILARITY AND CONGRUENCE ***

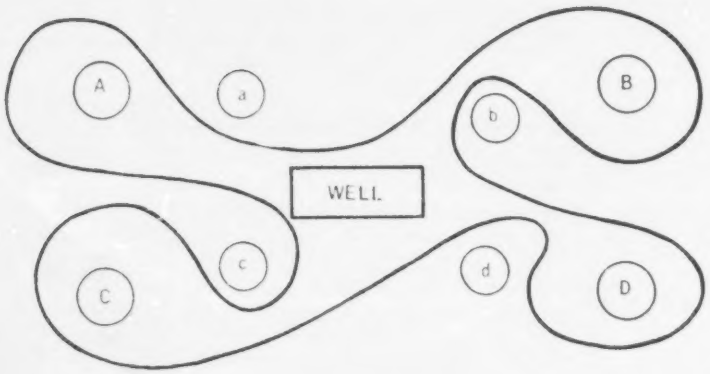
R-145 12 edges.

R-145 Only 1. After that, the box isn't empty.

*** SYMMETRY ***

S-123 The second player wins. The method is to break the circle of petals into two symmetrical semicircles. Whether the first player takes 1 petal or 2, the second player must answer by taking 2 or 1 respectively from the opposite end of the diameter, breaking the array into two blocks of 5 petals each. The second player then continues to restore the symmetry each time it is broken by the first, and consequently must draw the last petal.

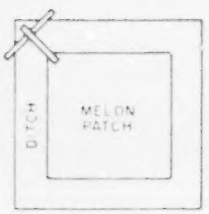
S-126




*** "MONEY" AND "TIME" ***

T-3 18 inches. Only 3 inches were lost on each yard in measuring 18 feet, or 6 yards, with the 33 inch stick, but nothing was lost in measuring the last two feet. Therefore, the total loss to the customer was 6 x 3 = 18 inches.

T-5




T-17 40 posts.

T-83 The number of days that will elapse before the ships leave New York on the same day is the smallest number that contains all the numbers 12, 16, and 20 as factors; that is, the least common multiple. Since the least common multiple of 12, 16 and 20 is 240, the number of days that will elapse before the ships leave New York on the same day again is 240. Consequently, the first ship makes 240 ÷ 12 = 20 trips, the second ship makes 240 ÷ 16 = 15 trips, and the third ship makes 240 ÷ 20 = 12 trips.

T-10 33 seconds. It is the interval between the strokes which takes up the time.

T-15 There are only two different selections of 12 coins to total 49¢: either 4 pennies, 7 nickels, and 1 dime or 9 pennies, 1 nickel, 1 dime, and 1 quarter.

So Peter had the 1 quarter, and the coins each had were as detailed above.

T-11 It is tempting to give the "obvious" answer 12, but the true answer is 11, as will be found by counting the individual occasions.

1. Between 6:30 A.M. and 6:35 A.M.
2. Between 7:35 A.M. and 7:40 A.M.
3. Between 8:40 A.M. and 8:45 A.M.
4. Between 9:45 A.M. and 9:50 A.M.
5. Between 10:50 A.M. and 10:55 A.M.
6. At noon.
7. Between 1:05 P.M. and 1:10 P.M.
8. Between 2:10 P.M. and 2:15 P.M.
9. Between 3:15 P.M. and 3:20 P.M.
10. Between 4:20 P.M. and 4:25 P.M.
11. Between 5:25 P.M. and 5:30 P.M.

T-12 If I write "x" checks a month, my present charge is

$25 + 10x$ cents.

Under the new arrangement I shall pay

$50 + 8x$ cents.

My saving per month is

$25 + 10x - (50 + 8x)$

or $2x - 25$

If I am to make a saving, this must be positive and $2x$ must be greater than 25. Dividing both items by 2

"x" must be greater than $12\frac{1}{2}$.

Hence, I must draw 13 or more checks a month to make a saving.

T-9 $141.00

T-10 $7.00.

T-100 There is no reason to expect the sum of the two columns to be equal. Experiments with other withdrawals and balances will clarify any erroneous thinking associated with this puzzle, which is exceptional in that the sum of the withdrawals is arranged to result to within $1 of the sum of the bank balance.

T-102 Carl paid nothing for borrowing a quarter for a week. It would take 5 years exactly for Jim's nickel to earn 1 cent interest at the rate of 4 per cent.

T-111 The clock which doesn't run at all is correct twice a day; the other is correct only twice a year. If a clock loses 4 minutes every day, and makes one complete revolution every 12 hours, it will take 180 days before the hands coincide again.

```
   24 hours
   60 minutes
 ------------
 1440 minutes in 24 hours
  360 times
 ---------
4 ) 1440
```

The clock needs to turn 360 times before the hands will coincide, or 180 days since the hour hand goes around twice in one day.

T-167 One fifty, two twenties, a five and four twos.

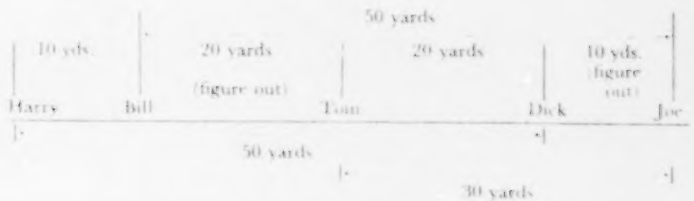
*** MEASUREMENT ***

V-31 It does make a difference. A submerged body displaces its volume; a floating body displaces its weight. Since a penny is denser than water, dropping it into the canoe will raise the water level higher.

V-46 They are the same height. Measure them.

V-8 Since the two pitchers contain equal quantitites of liquid at the beginning and at the end of the interchanges, the amount of milk in pitcher A must exactly equal the amount of water in pitcher B.

V-108



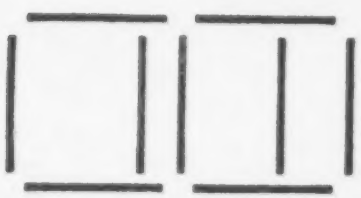

V-143 Length = 12 inches
Width = 10 inches
Height = 8 inches

*** NUMBER LINE ***

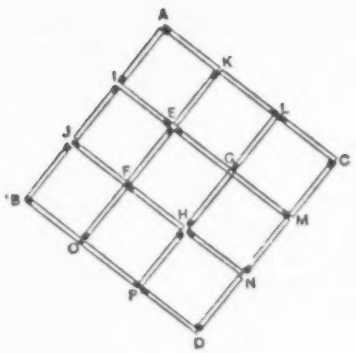
W-8 The five temperatures were 1, - 1, 2, - 2, and 3.

*** CONSTRUCTIONS ***

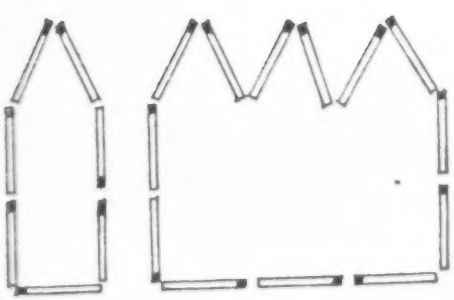
X-27



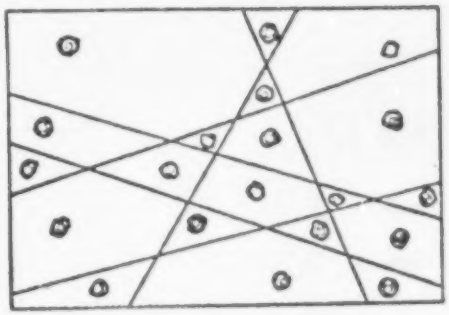
X-73 You now have nine small diamonds, and five large ones made up like this: ABCD; AJHL; IBPG; KFNC; EODM




X-81

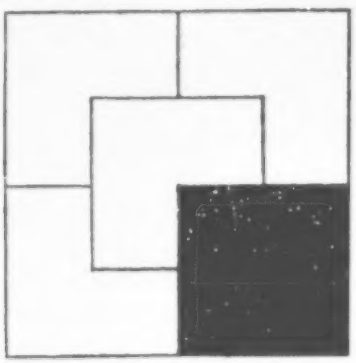


X-119 Bed of Roses

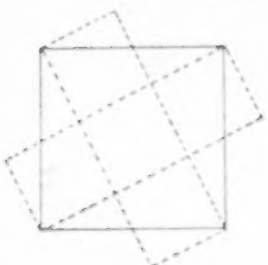


X-64 There are 60 vertices and 62 faces.

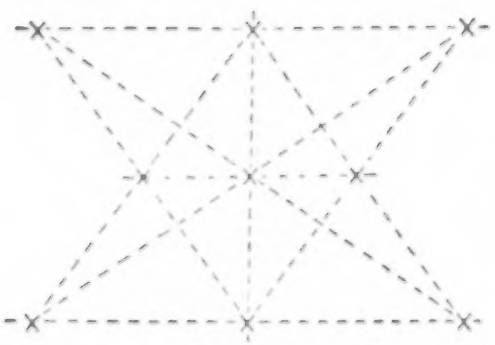
X-91 Square Deal



X-170 Squaring the square.



X-17



Old Prime had ten lines of trees.

Z-90 38% of all the students went to the game.

Z-71 Three socks. If the first sock drawn is blue and the second is black, then the third must match either one of the first two, since it must be either blue or black.

Z-105 THE NECKLACE. No general formula exists which can avoid the necessity for writing out the possible patterns and calculating separately how many different designs each represents. Here is the complete tabulation.

| | Pattern | Choice of Colors | Permutation of Colors | Product |
|---|---|---|---|---|
| 1 color | AAAAA | 4 | 1 | 4 |
| 2 colors | AAAAB | 6 | 2 | 12 |
| | AAABB | 6 | 2 | 12 |
| | ABABB | 6 | 2 | 12 |
| 3 colors | AAABC | 4 | 3 | 12 |
| | AABAC | 4 | 3 | 12 |
| | AABBC | 4 | 3 | 12 |
| | AABCB | 4 | 6 | 24 |
| | ABCAB | 4 | 3 | 12 |
| 4 colors | AABCD | 1 | 12 | 12 |
| | ABACD | 1 | 12 | 12 |
| | | | | 136 |

Z-45 Complete Solution:

| | Played | Won | Lost | Tied | Goals for | Goals against |
|---|---|---|---|---|---|---|
| A | 2 | 2 | 0 | 0 | 7 | 1 |
| B | 2 | 0 | 1 | 1 | 2 | 4 |
| C | 2 | 0 | 1 | 1 | 3 | 7 |

Scores: A vs. B 2-0,
B vs. C 2-2,
A vs. C 5-1.

Z-47 Complete Solution:

A vs. B 0-0, A vs. D 1-0, C vs. D 3-5.
A vs. C 1-0, B vs. D 4-3,

Z-113 Comparing numerators shows that the chance is about 3 to 1 in favor of success on the first try.

Z-4 (a) Boris (16); (b) Charlie (20); (c) Abner (18)
For an even number of tosses, the possibility of the same number of heads and tails being tossed reduces the probability of one being tossed more than the other. The higher the number of even tosses, the lower the probability of the same number of heads and tails. Therefore, the one least likely to have tossed more heads than tails is the one with the lowest number of even tosses.

Z-44 Since there is only one winner, there must be 52 losers requiring 52 matches.

Z-49 It is tempting to say that since the chance of a head when a single coin is tossed is a half, the chance of two heads out of four coins tossed is a half. This is not so.
Each of the four coins is equally likely to fall head or tail so that there are 2 x 2 x 2 x 2 = 16 equally likely results of tossing the four coins at once. These can be readily written down:

| | | |
|---|---|---|
| (1) H H H H | (7) H T T H | (12) T H T T |
| (2) H H H T | (8) H T T T | (13) T T H H |
| (3) H H T H | (9) T H H H | (14) T T H T |
| (4) H H T T | (10) T H H T | (15) T T T H |
| (5) H T H H | (11) T H T H | (16) T T T T |
| (6) H T H T | | |

Numbers 4, 6, 7, 10, 11 and 13 are the only cases where exactly two heads and two tails occur. Hence, two heads and two tails may be expected 6 times in 16 throws and the probability of them occurring is 3/8.

## *** DECIMALS ***

AA-86 The cork cost $0.05; the bottle cost $1.05.

AA-86 5 + 5 - 5 + .5 + .5 = 6.

AA-88

| | |
|---|---|
| 14 sheets of paper | $0.07 |
| 8 pens | .08 |
| 2 pencils | .05 |
| 1 eraser | .05 |
| 25 articles | $0.25 |

AC-4 A very simple puzzle!
Only a Lemon can say that he's not White; for a White, if he said so, would be lying; and an Orange would be telling the truth. So Mr. White is the Lemon; and Mr. Lemon, who says he is not a Lemon, must be the White. Hence he tells the truth about Mr. Yellow.
Mr. Yellow is a White.

AC-18 Mary's "generosity" is overrated. When John has scored 12 points, Mary should, at worst, have scored 3. Now they change over. Mary should score 18 points while John is scoring, at most, 7.
Mary should win by 2 points at least.

AC-23 You have to consider every possibility. If you draw 8 marbles, the stipulated conditions will probably be fulfilled, but "probably" is not good enough. To draw 8 marbles may leave all 8 blue ones, and only two of each of the other colors. To draw 7, however, covers all contingencies.
Seven marbles should be drawn.

AC-33 So the rider of Lambkin was Pipkin, who owns the horse Jorkin.

AC-35 The fourth speaker must be a Washington.

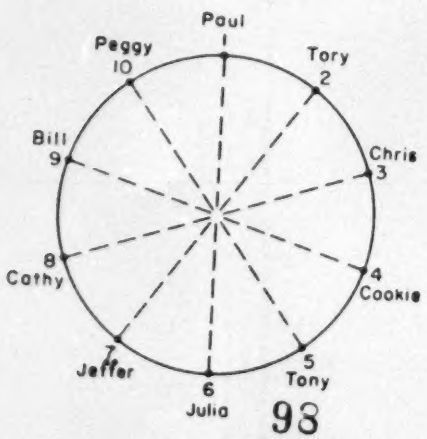
AC-175
(1) A and his wife go over, and A returns.
(2) B's and C's wives go over, and A's returns.
(3) B and C go over, and B and his wife return.
(4) A and B go over, and C's wife returns.
(5) A's wife and B's wife go over, and C returns.
(6) C and his wife go over.

AC-178 Although the tourist did not understand what the first man said, he realized that this man had to say, "I am an Omega," for, if he were an Alpha, he would have lied and said that he was an Omega and, if he was an Omega, he would have told the truth and said that he was an Omega. Hence, when the second man stated that the first man had said he was an Omega, he was telling the truth. Therefore, he must be an Omega, and since he said that he and the first man are both Omegas, this must be the truth. It immediately follows the third man lied, so he is an Alpha. Thus, the tourist hired the second man.

AC-179 While the president was marking the foreheads of each of the men, Joe reasoned as follows: The president cannot mark us all white, for if he did, no one would stand up when the lights are turned on, and we shall immediately know that we are all marked white. He must mark at least two of us black, for if he marks only one of us black, then one man will see no black marks; hence, he will remain seated and immediately realize his mark is black. Moreover, if the president makes only two black marks, then, when we all stand up, one of the two bearing black marks will conclude at once that he has a black mark, for otherwise, since he sees one white mark, there would be less than two black marks and, in that case, not all of us will stand up.

That is to say, if the president marks two men black and one white, the test will not be the same for each man, hence it will not be a fair test. Therefore, all marks on our foreheads must be black. Consequently, as soon as the lights went on, and they all stood up, Joe said, "I have a black mark."

AC-181 Assume that Paul sits at seat 1, and the boys and girls sit alternately. Hence the boys may sit on seats 3, 5, 7, and 9, and the girls sit on seats 2, 4, 6, 8, and 10. Since Tony and Chris sit next to Cookie, Cookie must be between Tony and Chris. Moreover, since Bill wants to sit next to Cathy and Cathy wants to sit next to Jeffer, then Cathy must sit between Bill and Jeffer. It follows that neither Cathy nor Cookie may sit on seats 2 or 10, but they may sit on seats 4, 6, or 8. If Cookie sits on seat 6, then Cathy cannot sit on seats 4 or 8, for she sits between Bill and Jeffer, and seats 7 and 5 will both be occupied. Similarly, Cathy cannot sit on seat 6. Hence, Cookie and Cathy may sit either on seats 4 or 8. Assume that Cookie sits on seat 4, then Peggy cannot sit on seats 2 or 6, for she would be next to Tony or Chris and she does not want to sit next to them; hence, she must sit on seat 10. Now Tory cannot sit on seat 6 for she must be opposite Jeffer, so she must sit on seat 2. It follows that Jeffer must sit on seat 7, opposite Tory. Cathy must sit on seat 8, and Bill on seat 9. Julia must then sit on seat 6 and either Tony or Chris on seat 5. If Tony sits on seat 5, then Chris must sit on seat 3. If Cookie sits on seat 8, the reader may check that the relative positions will be the same, but in a counterclockwise direction.



AC-189 Pete, Dan, and Sid are the respective husbands of Cora, Flo, and Jill.

AC-188 Morris is the physicist, Howe is the pianist, and Clark is the philosopher.

AC-189 Bill, Sam, and Jack are the respective husbands of Joan, Becky, and Grace.

AC-189 Ennis, Baker, and Cronin are the controller, manager, and treasurer, respectively.

AC-8 Kibi was a girl, the mother spoke first, and the father second.

AC-12 A, B and E are wearing black discs.
C and D are wearing white discs.

AC-21 Nineteen steps, in this order: 1, back to bank, 1, 2, 3, back to 2, 3, 4, 5, back to 4, 5, 6, 7, back to 6, 7, 8, bank, back to eight.

AC-45 It took 11 journeys. First the two sons crossed, then one came back. Then Mr. Smith crossed, and the second son returned. Then both sons crossed again, and one came back. Next, Mrs. Smith went over and the second son came back. Again, both sons went over--and finally, one of them came back for the basket.

AC-9 CAFE AU LAIT--The greater the temperature difference between the liquid and the surrounding air, the faster will heat be lost. Hence, the cup to which the cream is added later will lose heat faster and reach drinking temperature first.

AC-19 One solution would be as follows. Fill 11-oz. measure and then fill 5-oz. measure from it. Empty 5-oz. measure into the bottle, and fill it again from 11-oz. measure. Empty 5-oz. measure into the bottle, and then pour the remaining 1 oz. into it from the 11-oz. measure. Finally, fill the 11-oz. measure from the bottle, so leaving 12 oz. in the bottle.

AC-31

| | | |
|---|---|---|
| Alf | Bottle-Washer | $378 per month |
| Bert | Door-Shutter | 315 per month |
| Charlie | Door-Knob Polisher | 305 per month |
| Duggie | Worker | 310 per month |
| Ernie | Welfare Officer | 300 per month |
| Fred | Sweeper-Upper | 315 per month |
| George | Door-Opener | 248 per month |

AC-43 There were seven in the car--a boy and his two sisters, their mother and father, and the father's parents.

AC-141 Tom was the guilty one. Only one boy is lying; it can't be Dick or George because Tom and Harry contradict each other which means one of them is lying. And if George is telling the truth, the guilty party is Tom.

AC-192 The father-daughter relationship and their respective home states are: William, Dorothy, New York; Robert, Pat, Massachusetts; Thomas, Mary, New Jersey; Edward, Helen, Connecticut; Arthur, Joan, Pennsylvania; and Carl, Nancy, Virginia.

AC-6 Smith is between 30 and 40.
He is a member of Yonkers Youngsters.
He is not a member of Iowa Idlers.
He may not enter the Taunton Temple of Torpor.

References


1. Abraham, R. M. Diversions and Pastimes. New York: Dover Publications, Inc., 1964.

2. Doubleday, Eric. Test Your Wits, Volume III. New York: Ace Publishing Corp., 1972.

3. Emmet, E. R. Puzzles For Pleasure. New York: Emerson Books, Inc. 1972.

4. Friedland, Aaron J. Puzzles in Math and Logic. New York: Dover Publications, Inc., 1970.

5. Frohlichstein, Jack. Mathematical Fun Games and Puzzles. New York: Dover Publications, Inc., 1967.

6. Gardner, Martin. Mathematical Puzzles and Diversions. New York: Simon and Schuster, 1958.

7. Hart, Harold H. Grab a Pencil. New York: Hart Publishing Co., 1972.

8. Hunter, J. A. H. Hunter's Math Brain Teasers. New York: Bantam Books, 1965.

9. Hunter, J. A. H. Fun with Figures. New York: Dover Publications, Inc., 1956.

10. Kaufman, Gerald L. The Book of Modern Puzzles. New York: Dover Publications, 1954.

11. Kohl, Herbert R. Math and Writing Games in the Open Classroom. New York: Vintage Books, 1974.

12. Litton Industries, Inc. The Best of Problematical Recreations. Beverly Hills, California: Litton Industries, Inc., 1964.

13. Longley-Cook, L. H. Fun with Brain Puzzles. Greenwich Conn.: Fawcett Publications, Inc., 1965.

14. Longley-Cook, L. H. Work This One Out. Greenwich, Conn.: Fawcett Publications, Inc. 1960.

15. Luzzatto, Jack. Fun and Games with a Pencil. New York: Bantam Books, 1968.

16. Meyer, Jerome. Puzzle Quiz and Stunt Fun. New York: Dover Publications, Inc., 1956.

17. Meyer, Jerome S. FUN with Mathematics. Greenwich, Conn.: Fawcett Publications, 1966.

18. Mira, Julio A. Mathematical Teasers. New York: Barnes and Noble Books, 1970.

19. Mott-Smith, Geoffrey. Mathematical Puzzles. New York: Dover Publications, Inc., 1954.

20. Phillips, Hubert. My Best Puzzles in Mathematics. New York: Dover Publications, 1961.

21. Phillips, Hubert. My Best Puzzles in Logic and Reasoning. New York: Dover Publications, 1961.

22. Simon, William. Mathematical Magic. New York: Charles Scribner's Sons, 1964.

23. Sumners, George J. Test your Logic. New York: Dover Publications, Inc., 1972.

24. Taylor, John L., Walford, Rex. Simulation in the Classroom. England: Penguin Books, 1972.

25. Tocquet, Robert. The Magic of Numbers. Greenwich, Conn.: Premier Books, 1965.

26. Wescott, Alvin M. Creative Teaching of Mathematics in the Elementary School. Boston: Allyn and Bacon, Inc., 1967.

# APPENDIX C

## ANNOTATED BIBLIOGRAPHY OF SELECTED BOOKS AND MATERIALS



103

ANNOTATED BIBLIOGRAPHY OF SELECTED BOOKS AND MATERIALS

1. Adler, Irving. The Giant Golden Book of Mathematics. New York: Golden Press, 1958.
   Tells the reader about the extraordinary things that come from the use of the mathematics that we commonly study in elementary grades. Deals with practical, as well as esthetical, considerations of mathematics.

2. Barr, Donald. Arithmetic for Billy Goats. New York: Harcourt, Brace & World, Inc., 1966.
   An unusual approach to two-digit or binary counting. Goats have two front hoofs instead of ten fingers as we do, and the process of computing and computer arithmetic is introduced to the reader as William Gruff develops a way to keep a tally of his corncobs to prove that his greedy Uncle Max is stealing them. Should enhance mathematical appreciation and understanding. Attractive illustration. Intermediate grades.

3. Bendick, Jeanne, and Levin, Marcia. Take Shapes, Lines and Letters. New York: Whittlesey House, McGraw-Hill, 1962.
   Discusses the relationship of shapes, lines and letters to mathematics. Relates these ideas to nature, art and music. Clear, concise and well illustrated. Makes the subject simple and exciting.

4. Bendick, Jeanne. Shapes. New York: Franklin Watts, Inc., 1968.
   What is a shape? Is it round? Squiggly? Square? Pointy? Write your name. Does this have a shape? What do shapes do for us? Explores this fascinating world of geometry and topology in a most unusual and highly entertaining way.

5. Bendick, Jeanne, and Levin, Marcia. Take A Number. New York: Whittlesey House: McGraw Hill, 1961.
   The book explains to the young reader numbers, numeration systems, rules about numbers, bases, binary systems and computers, fun with numbers, special numbers, and reasons for numbers.

6. Corcos, Lucille. From Unsgah 1 to Oyalee 10. New York: Pantheon Books; Random House, 1965.
   An amusing counting book for young children using words and numbers of the Mohawk Indian Tribe.

7. Cuisenaire Rods. New Rochelle, New York: Cuisenaire Company of America, Inc., 12 Church Street (10805).
   Kits use rods of different lengths and colors to teach the fundamentals of mathematics such as multiplication, prime numbers, measurement, etc. The rods are intended to be used as devices for inventive mathematical activity in which creative expression is encouraged.

   A text, Using the Cuisenaire Rods, as well as several other publications, is available.

8. Diggins, Julia E. String, Straightedge, and Shadow. New York: Viking Press, 1965.
   Using these three wonderful tools, ancient men discovered the ideas and construction of elementary geometry more than twenty centuries ago. That is the subject of this story.

9. Emberley, Edward. The Wing on a Flea. Boston: Litrle, Brown and Company, 1961.
   Amusing imaginative picture book about triangles, rectangles and circles, and their relation in the concrete world.

10. Exploring Mathematics on Your Own. St. Louis: Webster Publishing Company, 1961.
    A series of enrichment booklets tailored for students who want to go beyond the textbook. Designed for use by upper grades and Junior High Schools. Titles include: Sets, Sentences, and Operations; The Pythagorean Theorem; Invitation to Mathematics; Understanding numeration Systems; Fun with Mathematics; Number Patterns; Topology--the Rubber-Sheet Geometry; The World of Statistics; Short Cuts in Computing; The World of Measurement; Adventures in Graphing; All about Computing Devices.

11. Feelings, Muriel. Moja Means One--Swahili Counting Book. New York: Dial Press, 1971. (Caldecott Winner)
    This is a very informative, cultural, easy to read counting book which reveals to children a foreign culture. As indicated by the Caldecott award, the illustrations are outstanding. The book also makes an attempt to make black students aware of their origins and native language.

12. Feravolo, Rocco. Wonders of Mathematics. New York: Dodd, Mead & Co., 1963.
    Discussing various aspects of mathematics from early beginnings to present day. Includes such topics as numbers, measurement, geometry, time, and some activity problems.

13. Freeman, Mae. Finding out about Shapes. New York: McGraw Hill, 1969.
    The book deals with shapes using simple, easy-to-read language, and simple illustrations.

14. Franklin Mathematics Series. Pasadena, California: Franklin Publications, Inc.
    A series designed specifically to extend and enrich the mathematical experiences of boys and girls in grades 3-8. Titles include: Learning about Measurement, Mathematics around the Clock, Patterns and Puzzles in Mathematics, From Fingers to Computers, Probability: The Science of Chance, Mathematics: Man's Key to Progress (Book A & B), Learning to Fold - Fold to Learn, Mirror Magic, Paper and Pencil Geometry, Making and Using Graphs and Nomographs.

15. Gardner, Martin. The Numerology of Dr. Matrix. New York: Simon & Schuster, 1967.
The "fabulous feats and adventures" of Dr. Matrix in his travels throughout the United States and his encounters with number theory, slight of word, and numecological analysis. A delightful, if ficticious, master mind.

16. Hargrove, Clarence Ethel. The Elementary and Junior High School Mathematics Library. Washington, D. C.: National Council of Teachers of Mathematics, 1960.
An annotated bibliography of books designed to enrich the instructional program by providing sources of information and recreational reading.

17. Hoban, Tana. Count and See. New York: MacMillan Publishing Co., 1972.
Excellent photographs depicting numbers as things you find in the real world, such as a dozen eggs, a bag of peanuts, a set of candles. The book is constructed so that the written numeral is shown, then the same number of dots and then the picture. Only drawback might be that the numbers skip from fifteen to twenty, then thiry, forty, fifty, one hundred.

18. Hoban, Tana. Look Again. New York: MacMillan Publishing Co., 1971.
Deals with how perceptions can differ among people. The construction of the book allows part of a photgraph to show through a small square, and turning the page reveals the entire picture. Excellent photography makes this an exciting book.

19. Hoban, Tana. Push, Pull, Empty, Full, A Book of Opposites. New York: MacMillan Publishing Co., 1972.
Pictures depict opposites and bring concepts to real life.

20. Hogben, Lancelot. The Wonderful World of Mathematics. Garden City, New York: Garden City Books, 1955.
Historical background of mathematics from the caveman to the industrial revolution. Many colorful illustrations.

21. Jonas, Arthur. New Ways in Mathematics. Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1962.

22. More New Ways in Mathematics. Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1964.
Interesting selected topics in elementary mathematics, both historical and contemporary.

23. Kennedy, Leonard M. Models for Mathematics in the Elementary School. Belmont, California: Wadsworth Publishing Co., Inc., 1967.
Descriptions of learning aids that may be made by a teacher of children for use in mathematics programs in the elementary school. Contains both "manipulative" and "visual" devices.

24. Kenyon, Raymond G. I Can Learn about Calculators and Computers. New York: Harper and Row, Publishers, 1961.
The book contains a brief history of numbers, calculators, and computers. It teaches how a modern computer works and then gives instructions for several calculators and computers you can build yourself. The book concludes with a list of terms used by computer scientists, and tells where you can obtain further materials on the subject.

25. Lauber, Patricia. The Story of Numbers. New York: Random House, 1961.
Without numbers, the world we know could not exist. How can we say this? This interesting book clears up the fascinating answer to this question and others about our world of numbers.

26. Lemke, Horst. One Times One. New York: Franklin Watts, Inc., 1968.
Simple counting picture book that teaches multiplication using repeated addition and division using repeated subtraction.

27. Lewis, Alfred. The New World of Computers. New York: Dodd Mead and Co., 1965.
Tells how this technological revolution got started and where it is headed. Illustrates the important role computers play in communication, school report cards, predicting election results, etc. Many fascinating photographs.

28 Madison Project Materials. Danbury, Connecticut: Mathematics Media Division, H. & M. Associates.
Kits of materials designed for independent exploration of mathematical topics. Materials and guided suggestions are included: centimeter blocks, discs, geobard, peg game, tower puzzle, and weights and springs.

29. Mathematics Enrichment. New York: Harcourt, Brace & World, Inc.
A series of fine books. The first three books--Programs A, B, & C--each deal with the concepts of sets, geometry and numberation and use the method of programmed instruction. Books D & E treat such topics as numeration systems, exponents, the slide rule, angular measurement and sets. Intended to promote intuitive understanding of arithmetic

30. Morse, Samuel French. Sea Sums. Boston: Little Brown and Co., 1970.
A beautiful poetry book incorporating numbers relating man to himself and to nature.

31. Ruchlis, Hy, and Engelhardt, Jack. The Story of Mathematics. Irving-on-Hudson, New York: Harvey House, Publishers, 1958.
A book that explains geometry to the young reader. Relates the mathematical world to shapes, designs, solid figures, and games and puzzles. Many helpful, easily understood illustrations.

32. Russell, Solveig Paulson. Lines and Shapes: A First Look at Geometry. New York: Henry Z. Walck, Inc., 1965.
A book that explains geometry by relating shapes and lines to objects in the concrete world. Very accurate illustrations clearly show the shapes.

33. Russell, Solveig Paulson. One, Two, Three, and Many: A First Look at Numbers. New York: Henry Z. Walck, Inc., 1970.
The book discusses the history of numbers, their origins and uses, and the development of machines to use numbers.

34. Schaaf, William L. Recreational Mathematics. Washington, D. C.: National Council of Teachers of Mathematics, 1963.

35. Simon, William. Mathematical Magic. New York: Charles Scribner's Sons, 1964.
A collection of entertaining tricks, stunts, and effects which can be performed by using numbers or objects such as calendars or playing cards, and which are based on numerical principles.

36. Smith, David Eugene. Number Stories of Long Ago, Washington, D. C.: The National Council of Teachers of Mathematics, 1962.
Intended for supplementary reading in the elementary school. Written in nontechnical language with an effort to connect the stories to human history.

37. Smith, David Eugene, and Linsburg, Jekuthiel. Numbers and Numerals. Washington, D. C.: The National Council of Teachers of Mathematics, 1965.
The story of numbers, telling how they came into use and what the first crude numerals, or number symbols, meant. Includes interesting facts about measures, adding and subtracting, and number superstitions.

38. Stonaker, Francis Benson. Famous Mathematicians. Philadelphia: J. B. Lippincott Co., 1966.
Stories of outstanding mathematicians through the centuries, including Euclid, Archimedes, Descartes, Newton, Gauss and others, plus several geniuses from our century. Written in terms clearly understandable to young readers.

39. Walter, Marion I. Boxes, Squares and Other Things. Washington, D. C.: National Council of Teachers of Mathematics, 1970.
A unit describing an experience in informal geometry that is based mostly on work with construction paper and milk cartons. Can be used by large and small groups as well as by individuals.

40. A Young Owl Book. New York: Holt, Rinehart & Winston, Inc.
A series of colorfully illustrated books that would appeal to primary grade youngsters. Titles include: The Caterpillar Man, Counting Lightly, Delight in Number, Fun with the Calendar, If You Can Count to Ten . . ., Stretching Numbers, Twenty White Horses, A Book of Division, Eleven and Three are Poetry, Ten Pennies for Candy, Adding: A Poem, One, Two, Three, Four, Four Threes are 12, What is Big?